Random Notes.

1982 Jan.

Mass relations — P. Rog. PRL 47, 1785 (82)

Flavor mixing should be $\neq 0$ at ~~grand~~ unification

Glashow PRL 45, 1914 (80)

Weinberg, Gauge Th. & Mod. Field Th. MIT press 76.

't Hooft et al Recent Dev. in G.T. Plenum NY 80.

Accelerated system

S. A. Fulling PR D 15, 2850 (75)

S. W. Hawking, Nature 248, 30 (74)
Comm Math Phys. 43, 199 (75)

J. B. Hartle & S. W. H. PR D 13, 2188 (76)

S. Midorikawa RIFP-456 (81)

Proton Decay

SU(5) Scheme

$U(1) = L - B = +1$ $\quad -1$ $\quad\quad U = -2$

$$\begin{pmatrix} \bar{d}_i \\ e \\ \nu \end{pmatrix}_L \quad \begin{pmatrix} d_i \\ \bar{e} \\ \bar{\nu} \end{pmatrix}_R \qquad (u_i\ d_i\ \bar{u}_i\ \bar{e}\ \not{\bar{\nu}})_L \sim (\bar{e}d_i,\ \bar{\nu}d_i\ d_i d_j\ \bar{e}\bar{\nu})\ \overline{10}$$

$$(\bar{u}_i\ \bar{d}_i\ u_i\ e\ \not{\nu})_R \sim (e\bar{d}_i\ \nu\bar{d}_i\ \bar{d}_i\bar{d}_j\ e\nu)\ 10$$

$U = +2$

$5 \qquad \bar{5}$

X: $\bar{d}_L \leftrightarrow e_L$ $\quad \Delta Q = \mp 4/3$ $\quad d_L \leftrightarrow \bar{e}_L$ $\quad u_L \leftrightarrow \bar{u}_L$

$\bar{d}_L \leftrightarrow \nu_L$ $\quad \Delta Q = \mp 1/3$ $\quad u_L \leftrightarrow \bar{e}_L$ $\quad d_L \leftrightarrow \bar{u}_L$

U(1) conservation: $\text{lepton}_L \# = \bar{d}_L \#$

$$2\,\text{lepton}_L = \text{lepton}_R = \bar{u}_R = \bar{d}_R = u_R$$

So $\quad 2l_R + l_L + \bar{d}_L + 2\bar{d}_R + 2\bar{u}_R + 2u_R$

$- 2\bar{l}_L - \bar{l}_R - d_R - 2d_L - 2u_L - 2\bar{u}_L$ = conserved.

If q_L and q_R mix $\rightarrow$ $\# q_L = \# q_R$

$3l + 3\bar{d} - 3\bar{l} - 3d$ = conserved. ?

Is this $L - B$?

proton $uud \rightarrow 0\,0\,e^+$ is allowed.

Selection rules due to mass

	U(1)	1		2	
$e_L \bar{e}_L$	or	e_L	$\longleftrightarrow$	e_R	$\Delta U = 1$
$\bar{d}_L d_L$		$\overset{1}{\bar{d}_L}$	$\longleftrightarrow$	$\overset{2}{d_R}$	〃
$u_L \bar{u}_L$		$\overset{-2}{\bar{u}_L}$	$\longleftrightarrow$	$\overset{+2}{u_R}$	$\Delta U = -4$

Mass matrix 5×5; $5\times\overline{10}$ $\quad \bar{5}\times 10$; $10\times\overline{10}$;

$$5\times\overline{10} = \square \times \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} = \underset{\bar{5}}{\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}} \oplus \underset{45}{\begin{smallmatrix}\square\square\\\square\\\square\end{smallmatrix}}$$

$$10\times\overline{10} = \begin{smallmatrix}\square\\\square\end{smallmatrix} \times \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} = \underset{\bar{5}}{\begin{smallmatrix}\square\\\square\\\square\\\square\\\square\end{smallmatrix}} \oplus \underset{45}{\begin{smallmatrix}\square\square\\\square\\\square\\\square\end{smallmatrix}} \oplus \underset{50}{\begin{smallmatrix}\square\square\\\square\square\\\square\end{smallmatrix}}$$

$$5\times 5 = \square\times\square = \underset{10}{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \oplus \underset{15}{\square\square}$$

Charge has to be conserved.

If only 5_L & $\bar{5}_R$ existed, only ν_L & $\bar{\nu}_R$ can mix!

$5\times\overline{10}$: $\quad M \sim 5$ then $\quad M$ must be a spurion $\sim \boxed{\nu}$

So

$$\boxed{\nu}\times\begin{pmatrix}\bar{d}\\ e\\ \nu\end{pmatrix}_L \rightarrow \begin{matrix}[\nu\bar{d}] \sim \\ [\nu e] \sim \\ [\nu\nu] \to 0\end{matrix}\begin{pmatrix}\bar{d}\\ e\\ \ \end{pmatrix}_R$$

$$\rightarrow \quad m_e = m_d \qquad m_u = 0$$

$10 \times \overline{10}$ If $M \sim 5$

$$\boxed{\nu} \times \begin{pmatrix} \bar{u} \\ \bar{d} \\ u \\ e \end{pmatrix}_R \rightarrow \begin{array}{l} [\nu \bar{u}] \sim [\nu e \bar{d}] \sim [d d] \sim \bar{u}_L \\ [\nu \bar{d}] \sim [\nu \nu \bar{d}] \rightarrow 0 \\ [\nu u] \sim [\nu \bar{d} \bar{d}] \sim [\bar{e} \bar{d}] \sim u_L \\ [\nu e] \sim [\nu e \nu] \rightarrow 0 \end{array}$$

Only $m_u \neq 0$.

From $U(1)$ viewpt, M_5 causes $\Delta U = \pm 1$ mod 5 !

± 1 ↓ e, d mass; 5 ↓ u mass.

Majorana ν mass $M_{15} \sim \boxed{\nu\nu}$. $\Delta U = \pm 2$

Dirac ν mass : one needs a singlet ν_R and

$$M_5 \sim \boxed{\nu}$$

$U(1)$ charge of $\nu_R = \pm 5 \sim 0$. $\Delta U = \pm 1$

o S. Nandi, A. Stern & E.C.G. Sudarshan DOE-ER-03992-472
Jan 82

Proton Decay can be rotated away if s_3 (third K-M angle) = 0.

o J. Ellis, Phenomenology of Unif. Gauge Theories
TH-3174-CERN Sept 81.

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o Disordered system: Level density
H. Neuberger IAS Jan 82.

I.M. Lifshitz Usp. Fiz. Nauk 83, 614 (64)
[Sov. Phys. Usp. 7, 549 (65)]

Level density at small energy ε for random scat. centers

$$n(\varepsilon) \sim \exp[-k\varepsilon^{-d/2}] \qquad k \propto \text{density}$$

J. Cardi J. Phys. C11, L321 (78)

∘ Replica technique

S.F. Edwards, J. Phys. C8, 1660 (75)

$$\langle x | \frac{1}{\varepsilon - H} | y \rangle = \int \prod_{j=1}^{n} D[\phi_j(z)] \; \phi_1(x)\, \phi_1(y) \; \exp\left[+\tfrac{1}{2} \phi_j (\varepsilon - H) \phi_j \right]$$

lim $n \to 0$.

$$\int e^{-\phi K \phi} = \frac{1}{\sqrt{K}}. \qquad \int \phi^2 e^{-\phi K \phi} = \frac{1}{K^{3/2}}$$

∘ Supersymmetric particles

M. Henneaux & C. Teitelboim

U. Austin 1982

∘ Sakurai's question : γ-ρ mixing $\sim e/2g = \gamma_\rho$

$\gamma_\rho^2 \sim 1/300$

γ-~~W~~Z_0 mixing $\sim .23$ why so large ?

$\gamma_{Z_0} \sim e/g'$ why g' small ?

∵ ~~hadronic~~ energy scales 1 Gev → 1 Tev

$m_\rho \rightarrow m_W$ small

Then the question : why m_W small ?

So this is begging the question .

Bound on m_W : From mixing matrix $L = \begin{pmatrix} k^2 & \gamma k^2 \\ \gamma k^2 & k^2 - m_W^2 \end{pmatrix}$

MPI-PAE/PTh 22/82

The current-current term as $k^2 \to 0$: $= -\dfrac{(g-\gamma e)^2}{m_W^2}$

$$\rightarrow \frac{\gamma e}{g} \equiv \sin^2\theta$$

pole of propagator . $m_Z^2 = m_W^2/(1-\gamma^2)$, $\rightarrow \gamma \lesssim 1$

Also $g^2/m_W^2 = G_F$ fixed

$$\Rightarrow \gamma^2 = \sin^4\theta \, \frac{m_W^2 G}{e^2} < 1 \Rightarrow m_W \lesssim 170 \text{ Gev}$$

$(\sin^2\theta = 0.22)$

Blas Cabrera 3/28/82 Stanford preprint.

Flux estimate $f = 0.53 / m^2 d \, sr$

20 cm^2 4 turns 151 days of observation

1 event (+ 1 during occupation ?)

$f = 0.61 \times 10^{-9} / sec \, cm^2 \, sr$

mag. charge $g = \frac{1}{2e} \hbar c = 3.1 \times 10^{-8}$ cgs

Work / sec cm^3 $= fgB$

Energy / cm^3 $= B^2/8\pi$

Lifetime of mag field $\frac{1}{\tau} = \frac{fgB}{B^2/8\pi} = \frac{8\pi g f}{B} = 7.8 \times 10^{-7} \frac{f}{B}$

$f = 0.6 \times 10^{-9}$ → 4.7×10^{-10} sec^{-1}

$B = 10^{-6}$

$\tau \sim 2 \times 10^{9}$ sec 68 yrs.

On Liouville eq.

J. Liouville, J. Math. Pures Appl. 18, 71 (1853)

H. Bateman, Partial Diff. Eqs of Math Phys. (Dover, N.Y. 1944)

Instantons
- E. Witten, PRL 38, 121 (77)
- Chia Kwei Peng, Sci Sin. 20, 345 (77)
- L. Dolan PRD 15, 2337 (77)

Strings
- R. Omnès NP B149, 269 (77)
- B. Barbashov, V. Nesterenko & A. Chervyakov, Teo Mat Fiz. 40, 15 (79) (Theor Math Phys 40, 572 (79))
- A Polyakov PL 103B, 207 (81)

E. D'Hoker & R. Jackiw PRD (82)

Relation between SU(5) & SO(10)

Representation of SO(10)

Spinor $32 + \overline{32}$: in $\sigma^{(1)} \otimes \sigma^{(2)} \otimes \cdots \sigma^{(5)}$

Vectors V_α $\alpha = 1, \cdots 10$

V_1, V_2 : $(\sigma_1^{(1)}, \sigma_3^{(1)}) \times \sigma_2^{(2)} \times \cdots \sigma_2^{(5)}$

V_3, V_4 : $(\sigma_1^{(2)}, \sigma_3^{(2)}) \times \sigma_2^{(3)} \times \cdots \sigma_2^{(5)}$

$\vdots$

V_9, V_{10} : $(\sigma_1^{(5)}, \sigma_3^{(5)}) \times 1$

and $\sigma_2^{(1)} \times \sigma_2^{(2)} \times \cdots \times \sigma_2^{(5)} \equiv V_0$

$$L_{\alpha\beta} = [V_\alpha, V_\beta] \quad , \quad \{V_\alpha, V_\beta\} = 2\delta_{\alpha\beta}$$

$$[L_{\alpha\beta}, V_0] = 0 \rightarrow \text{Subspace } V_0 = \pm 1$$

Subgroup

A: $(L_{12}, L_{34}, \cdots, L_{9\,10}) = \underline{5}$, commuting generators

Ψ is in the space $\psi^{(1)} \otimes \psi^{(2)} \otimes \cdots \psi^{(5)}$ $(V_0 = \pm 1)$ $\in U(16)$

Permutations of the $\psi^{(i)}$'s effected by

$$P_{ik} = \frac{\sigma^{(i)} \cdot \sigma^{(k)} + 1}{2} \quad \text{not in } O(16)$$

But

B: $\begin{cases} L_{13} + L_{24}, & L_{14} - L_{23} \\ L_{15} + L_{26}, & L_{16} - L_{25} \\ \cdots \end{cases}$ $\quad 2 \times \binom{5}{2} = \underline{20}$

$A \oplus B$ form $U(5)$

U(1) generator $= \sigma_2^{(1)} + \sigma_2^{(2)} + \cdots \sigma_2^{(5)} = Q$

Q is correlated with V_0:

	$\sigma_2^{(1)}$,	$\sigma_2^{(2)}$,	..		$\sigma_2^{(5)}$		Q	#	
$V_0 = +1$:	+	+	+	+	+		5	1	
	−	−	+	+	+	etc	1	$\overline{10}$	
	−	−	−	−	+	etc	−3	5	□
$V_0 = -1$	−	−	−	−	−		−5	$\overline{1}^{*}$	
	+	+	−	−	−	etc	−1	10	日
	+	+	+	+	−	etc	3	5	

So $V_0 = \pm 1$ are conjugates.

Charge conjugation $\Psi \to V_0 \Psi^+$

$V_1, V_2 \to -1 \times$

$V_3, V_4 \to +1 \times$

$V_5, V_6 \to -1 \times$

..

$V_9, V_{10} \to -1 \times$

$V_0 \to -1 \times$

Another way of correspondence:

$$V_1 \pm i V_2 \rightarrow \psi_1, \psi_1^+$$

$$V_3 \pm i V_4 \rightarrow \psi_2, \psi_2^+$$

$$\cdots$$

$$V_9 \pm i V_{10} \rightarrow \psi_5, \psi_5^+$$

$$L_{12}, L_{34}, \cdots L_{910} \rightarrow \psi_1^+\psi_1, \psi_2^+\psi_2, \cdots \quad 5$$

Other: $\psi_i^+\psi_j \quad i \neq j \quad 20$ } $\underline{25}$

So $\psi_i^+\psi_j \quad i = 1, \cdots 5$ form $U(5)$

Adjoin $\psi_i \psi_j, \quad \psi_i^+ \psi_j^+ \rightarrow$ ~~$U(10)$~~ $SO(10)$

~~$\sum_i \psi_i^+\psi_i$ = fixed $\rightarrow SO(10)$~~

[This follows from the Pauli-Gürsey group of

$\bar{\psi} \gamma_\mu \partial_\mu \psi + m \psi^+ \psi$]

If $m = 0$, with Dirac spinors, one has $U(10)$.

$$U(10) \nearrow U(1)_{chiral} \times SU(10) \rightarrow SO(10) \searrow U(5)$$

$$U(10) \searrow U(5)_L \times U(5)_R \nearrow U(5)$$

Another sequence :

$$SO(10) \rightarrow SO(6) \times SO(4)$$

or $SU(4)$

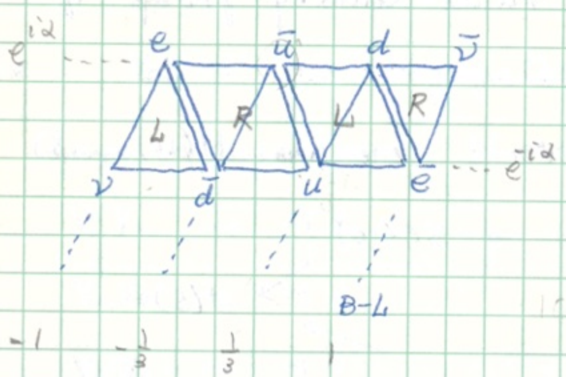
$$SO(6) \text{ or } SU(4) \downarrow SU(3)_c \qquad SO(4) \downarrow SU(2)_f$$

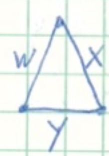
Representations are strange combinations

$(\bar{d}, \bar{e})_L$ $\qquad$ $\binom{e}{\nu}_L$, $\qquad$ $(u, d, \bar{u})_L$

$\underline{3}_c \oplus \underline{1}_f$ $\qquad$ $\underline{2}_f$ $\qquad$ $(\underline{3}_c \otimes \underline{2}_f) \oplus \underline{3}_c$

SU(5) periodic table





Chiral anomaly problem.

Majorana mass

1. $\gamma\cdot p\,\psi \sim D(\tfrac{1}{2},\tfrac{1}{2}) \otimes D(\tfrac{1}{2},0) \sim D(0,\tfrac{1}{2})$

If $\gamma\cdot p\,\psi = \lambda\,\psi^c$, $(\gamma\cdot p)\,\psi^c = +\lambda^*\psi$ $\qquad \psi^c = \rho_2\sigma_2\psi^*$

$\Rightarrow \quad p^2 = +\lambda\lambda^* \qquad (p^2 = p_0^2 - \mathbf{p}^2)$

If $\gamma\cdot p\,\psi = \lambda\,\gamma_5\,\psi^c$

$\gamma\cdot p\,\psi^c = -\lambda^*\gamma_5\psi$ $\quad \rightarrow \quad p^2 = +\lambda\cdot\lambda^*$

2-compt

If $(p_0 + \sigma\cdot p)\,\psi = \lambda\,\psi^c = \lambda\,\sigma_2\psi^*$

$(p_0 - \sigma\cdot p)\,\psi^c = \lambda^*\psi$ $\qquad p^2 = \lambda\cdot\lambda^*$

N.B. In writing : $(p\,\psi)^* = -p\,\psi^*$

$\therefore \quad (i\nabla\psi)^* = -i\nabla\psi^*$

The norm is not $\psi^*\psi$.

$j_{\mu 5} = \bar\psi^c\gamma_\mu\gamma_5\psi$, $\partial_\mu j_{\mu 5} = 0$, $\partial_\mu j_{\mu 5}^\dagger = 0$

2. $\gamma\cdot p\,\psi = \lambda\,\psi$ $\qquad$ Dirac mass

$j_{\mu 5} = \bar\psi\gamma_\mu\gamma_5\psi$ $\qquad \partial_\mu j_{\mu 5} = -2i\,\lambda\,\bar\psi\gamma_5\psi$

3. $j_{\mu 5} \equiv \bar{\psi}\gamma_\mu\gamma_5\psi$ $\qquad$ $j_0 = \psi^* \rho_1 \psi = \langle \sigma \cdot v \rangle = 2hv$

if h = spin

$$\partial_\mu j_{\mu 5} = -2im\bar{\psi}\gamma_5\psi$$

$$J_\mu = -A_\nu \overset{*}{F}_{\mu\nu}$$

$$\partial_\mu J_\mu = -\frac{1}{2} F_{\mu\nu} \overset{*}{F}_{\mu\nu} = +2E\cdot B$$

$$J_0 = -\vec{A}\cdot\vec{B} = \vec{A}\cdot(\vec{\nabla}\times\vec{A}) = -\sum_k i\hat{k}\cdot(a_k^\dagger \times a_k) = +hn$$

$$\vec{J} = -\varphi\vec{B} - \vec{A}\times\vec{E} = \vec{A}\times\vec{\pi} = \vec{\Sigma} \text{ (spin)}$$

So if $\partial_\mu j_{\mu 5} = \frac{g^2}{16\pi^2} F_{\mu\nu} \overset{*}{F}_{\mu\nu} \rightarrow -\frac{g^2}{8\pi^2}\partial_\mu J_\mu$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $\parallel$ $2\,\mathrm{tr}\, F\cdot\overset{*}{F}$

$j_{\mu 5} + \frac{g^2}{8\pi^2} J_\mu$ is conserved.

Instanton # is $\frac{g^2}{32\pi^2} F\cdot\overset{*}{F} = \frac{1}{2}\partial_\mu j_{\mu 5}$ = helicity

$$\parallel$$

$$-\frac{g^2}{8\pi^2} E\cdot B$$

[Schwinger, Adler, Bell-Jackiw formula:

$$\partial_\mu j_{\mu 5} = -\frac{e^2}{32\pi^2} F\cdot\overset{*}{F} = +\frac{e^2}{8\pi^2} E\cdot B \quad]$$

(32 → 16, 8 → 4)

↙ 2 s helicity $\qquad$ ↓ 2n by instantons.

Presence of monopoles ?

$$\partial_\mu J_\mu = -\partial_\mu A_\nu \overset{*}{F}_{\mu\nu} - A_\nu \partial_\mu \overset{*}{F}_{\nu\mu} = -\frac{1}{2} F\cdot\overset{*}{F} + g_m k_\nu A_\nu \quad ?$$

↙ mag. current.

In the θ-vacuum, one should use a total divergence,

$$\text{so} \quad \frac{g^2}{\cancel{32}\,16\,\pi^2}\,\theta\,\partial_\mu J_\mu = -\frac{g^2\theta}{32\pi^2}\,F\cdot\overset{*}{F} + \frac{g^2\theta}{16\pi^2}\,k\cdot A\,g_m$$

~~If $g\,g_m$~~ Here the def. of g is for $SU(2)$: $\frac{\tau}{2}g$. So the electric charge $e = g/2$ $\qquad e\,g_m = 2\pi$

$$\Rightarrow \quad \frac{g^2 g_m \theta}{16\pi^2} = \frac{e^2 g_m \theta}{4\pi^2} = \frac{e\theta}{2\pi}$$

SQUID + Josephson effect + Schrödinger's cat.

S. Nakajima — Sor. Kenk. 64, No 4. p 151 (82)
A.J. Leggett — J. de Phys. Coll. C6 (78) 1264;
PTP Suppl.
R. de Bruyn Ouboter + W. den Boer, Physica 98B, C (80) 185

$$H = \frac{1}{2C} p^2 + V(\phi)$$

$$V(\phi) = -\frac{I_c \phi_0}{2\pi} \cos\left(2\pi \frac{\phi}{\phi_0}\right) + \frac{1}{2L}(\phi - \phi_x)^2$$

$$\phi_0 = \pi/e$$

Derivation: Usual energy $\frac{1}{2} C V^2 + \frac{1}{2} L I^2$

$$V = -\oint E\, ds = +\int \dot{B}\, da = +\dot{\Phi} \to p = CV$$

Also $V = L\dot{I} \to \Phi - \Phi_0 = LI$

Tunnelling: unperturbed eigenfns for p^2 term are $e^{in\phi/\alpha}$. Tunnelling causes transitions $\to e^{i(n\pm1)\phi/\alpha}$

So the interaction $\propto \cos(\phi/\alpha)$