VPI 6/15

Sakita. Early Strings

1. Veneziano
2. KSV & zero slope limit → Scherk
3. Koba-Nielsen
4. Path Integral HSV
5. Light cone
6. Nambu-Susskind
7. Nambu-Goto
8. Polyakov
9. Multi-loop
10. Field th.

2. $g^2 = \alpha' \lambda^2$ $\alpha = \alpha^2(s - m^2)$ $\alpha' \to 0$

$$A(s,t) \to -\lambda^2\left(\frac{1}{s-m^2} + \frac{1}{t-m^2}\right) \quad : \lambda\phi^3 \text{ theory}$$

KSV 1969

3. Koba-Nielsen

$$x = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)} = \text{real} \quad 0 < x < 1$$

$$1-x = \frac{(z_1-z_4)(z_2-z_3)}{(z_1-z_3)(z_3-z_4)} \qquad \frac{dx}{x(1-x)} = \frac{dz_1\,(z_4-z_2)}{\underset{3}{(z_1-z_2)(z_4-z_1)}} = \frac{\prod dp_i}{\prod_i (z_{i+1}-z_i)}\frac{1}{d^3F}$$

$$d^3F = \frac{dz_2\,dz_3\,dz_4}{(z_2-z_3)(z_3-z_4)(z_4-z_2)}$$

$$s = -(k_1+k_2)^2, \quad t = -(k_3+k_2)^2 \qquad k_i^2 = \alpha_0^2/\alpha'$$

$$x^{-\alpha(s)}(1-x)^{-\alpha(t)} = \prod_i (z_{i+1}-z_i)^{\alpha_0} \prod_{i \neq j} (z_i - z_j)^{\alpha^{\frac{1}{2}} k_i \cdot k_j}$$

$$B = \int \frac{\prod dz_i}{d^3F} \prod |z_{i+1}-z_i|^{\alpha_0 - 1} \prod_{i\neq j} |z_i - z_j|^{\alpha' k_i \cdot k_j}$$

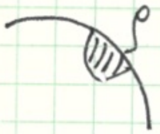
a) N-part ampl.
b) Mobius inv. $SL(2,R)$
c) $\alpha_0 = 1 \to$ conformal inv.
d) factorization Bardakci-Mandel, Gordon-Fubini-Venez. Nambu

4. Path $\int$al. HSV (1970)

$$|z_i - z_j|^{\alpha' k_i \cdot k_j} = \exp \sum_{i \neq j} \alpha' k_i \cdot k_j \ln |z_i - z_j|$$

↳ Greens fn.

$$\to \frac{1}{\pi E_i} \exp\left[-\frac{1}{2}\int d^2\xi \int d^2\xi' \, j(\xi)\, j(\xi')\, N(\xi,\xi')\right]$$



$$j^\mu = \sum k^\mu \rho(\xi)$$

↳ smearing

$$\rho_i = \sqrt{2\pi\alpha'}\, \delta(\xi_1 - \cos\theta_i)\, \delta(\xi_2 - \sin\theta_i)$$

$$E_i = \exp\left[\frac{k_i^2}{2\pi_i} \int d\xi \int d\xi' \rho_i \rho_i' \ln|z_i - z_i'|\right]$$

$$N = -\frac{1}{2\pi} \ln |z - z'||z - \bar{z}'|$$ Neuman b.c.

Gervais-Sakita: $1/\mathcal{L}_{N.G.}$ needed for integration measure.

→ First order form of Polyakov (Neveu)

Conformal gauge: FP → Ghost field to make it local

ψ

Fujikawa

Gervais - Sakita 1973

J. Gates. Nonlinear σ & sustr.

$D = 2.$

$$S = \frac{1}{2}\int d^2x \, \frac{1}{2} g_{ij}\, \partial^\mu \varphi^i \partial_\mu \varphi^j \qquad g(\varphi)$$

$M^2 \ni x^0, x^1$

$M^D \ni \varphi^i$ so $M^2 \to M^D$

Let $(\varphi^i)' = f^i(\varphi)$ $\quad (g^{ij}(\varphi'))' = g_{kl}(f)\, f^i_{,k} f^i_{,l}$

gen. coord inv $\qquad g_{ij} = V_i^a V_j^b \eta_{ab} \quad V(\varphi)$

SUSY. $\psi^{i\alpha}(x)$ Majorana $\qquad S' = i\frac{1}{2}\psi^i \not\nabla \psi^j_{g_{ij}} + \frac{1}{12} R_{ijkl} \bar\psi^i \psi^j \bar\psi^k \psi^l$

$$\nabla_\mu \psi^i = \partial_\mu \psi^i + \partial_\mu \varphi^k \Gamma_{kj}{}^i \psi^j \qquad \Gamma \text{ from } g_{ij}(\varphi)$$

ψ^i: vector in M^D $\qquad$ Freedman - Zumino

1st susy $\quad \delta\varphi^i = \bar\epsilon \psi^i, \quad \delta\psi^i = i\, \cancel{\partial}\varphi^i \varepsilon - \cdots \qquad N=1$ susy

Additional susy if: $\delta\varphi^i = (\epsilon \varphi^j) f_j^i(\varphi), \quad \delta\psi = \cdots$

if $\nabla_i f^j = 0, \quad f_i{}^j f_j{}^k = -\delta_i{}^k$

$\Rightarrow$ Kähler $\quad g_{ij} f_k{}^i f_l{}^j = g_{kl} \qquad N=2$

$z^i = \varphi^i + f_j{}^i \varphi^j$

$\bar z^i = \text{''} - \text{''}$

$$g_{AB} = \left(\begin{array}{c|c} 0 & g_{i\bar j} \\ \hline g_{\bar i j} & 0 \end{array}\right) \qquad g_{i\bar j} = \frac{\partial}{\partial z_i}\frac{\partial}{\partial \bar z_j} K(z, \bar z)$$

$$\delta\varphi^i = (\bar\epsilon^\mu \psi^j) f_{\mu j}{}^i(\varphi) \qquad \mu = 2, \cdots N$$

irreducible manifold of $M^d \Rightarrow N = 1, 2, 4.$

$$f_{Mi}{}^j f_{Nj}{}^k + f_{Ni}{}^j f_{Mj}{}^k = -2\,\delta_i{}^k \delta_{MN} \to \text{Clifford.}$$

$N=4$ susy $\quad$ hyperkähler manifold.

$f_{(1)}, f_{(2)}, f_{(3)}$

$D=4$: $N=1$ iff Kähler) scalar multiplets

$N=2$ .. Hyper

$D=6$:

Start from $D=4$. look at $(V_\mu(x)$ $N=2$

$\lambda^i(x),\ A, B)$

reduce to $D=2$ $v_i, C, D, \lambda^{\hat{i}}, A, B$

$\underset{2\ sc.}{v_i}$, $\underset{4}{\lambda^{\hat{i}}}$

after reduction, new terms

$$S_2 = S_1 + \int d^2x \left[\frac{1}{2} \underset{\text{antisym.}}{h_{ij}}\, \partial^\mu\varphi^i \underset{\varepsilon_{\mu\nu}}{\partial_\mu} \varphi^j + \frac{1}{12} R_{ijkl} (\bar\psi^i\gamma^3\psi^k)(\bar\psi^j\gamma^5\psi^l)\right]$$

$$= \int d^2x \int_0^1 d\tau\ T_{ijk}(\varphi)\, \frac{\partial\hat\varphi^i}{\partial\tau}\, \partial^\mu\hat\varphi^j \partial^\nu\varphi^k\, \epsilon_{\mu\nu}$$

if $\hat\varphi^i(x,0)=0$, $\hat\varphi^i(x,1) = \varphi^i(x)$

$T_{ijk} = (\partial_i h_{jk} + \partial_j h_{ki} + \partial_k h_{ij})$ torsion of M^D

(in $D=4$, $h_{AB}(A,B)\, F^A_{\mu\nu} \tilde F^B_{\mu\nu}$)

$$D=10: \quad \begin{cases} g_{ij}(\varphi) & B_{ij}(\varphi),\ \Phi(\varphi) \\ \psi_i(\varphi) & \lambda^i(\varphi) \end{cases}$$

$$\int d^2x \left[\Phi(\varphi) R_{(2)} \frac{1}{\alpha'} + \cdots \right]$$

so add $D=2$ SG.

susy field: $\eta(x,\theta) = \varphi^i(x) + \theta^\alpha \psi_\alpha{}^i(x) - \theta^2 F^i$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\,\theta (\not\partial)_{\alpha\beta}$$

$$S_1 = \int d^2x\, d^2\theta\ g_{ij}(\eta)\, D^\alpha\eta^i D^\beta\eta^j\, C_{\alpha\beta}$$

$$S_2 = \int \cdots\ h_{ij}\, D^\alpha\eta^i D^\beta\eta^j\, (\epsilon\gamma^3)_{\alpha\beta}$$

$$\int d^2x \, e^{-1} \, (e_k{}^{\mu}\partial_\mu \varphi^i)(e_\ell{}^{\nu}\partial_\nu \varphi^j)\, \eta^{k\ell} g_{ij}$$

$D=2$ SG : $(e_\mu{}^{\nu}(x), \psi_\mu, \ A)$ off-shell

So $D=10/D=2$ $\quad S = \int d^2x \, d^2\theta \, E^{-1} R_{(2)} \Phi(\eta)$

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afternoon. $\quad S_1 = \int d^2x \, g_{ij} \, D_\alpha \eta^i D_\beta \eta^j \, \epsilon^{\alpha\beta}$

$\qquad\qquad S_2 = \qquad h_{ij} (D_\alpha \eta^i)(D_\beta \eta^j)(C\gamma^3)^{\alpha\beta}$

$D=10$: $\quad \underline{V}_i{}^{\alpha}(\eta) \quad h_{ij}(\eta) \quad \Phi(\eta)$

$D=2$: $\quad (e_\mu{}^a(x) \quad \psi_\mu(x), \ A)$

$A \equiv (D_\alpha, \partial_\mu, M) \qquad D_\alpha = \dfrac{\partial}{\partial\theta^\alpha} + i\theta^\beta (C\not\partial)_{\alpha\beta}$

$M =$ generator of "spin" $(D=2)$

$$\{D_\alpha, D_\beta\} = i\partial (C\gamma^\mu)_{\alpha\beta}\partial_\mu \qquad [M, D_\alpha] = 0 \qquad [M, \partial_\mu] = 0$$

$\to \quad \exp[-K^\alpha D_\alpha - K^\mu \partial_\mu - KM] \equiv e^{-K}$

$\qquad (\eta^i)' = e^{-K}\eta^i \quad : \quad \delta\eta^i = K^\mu \partial_\mu \eta^i$ or $\eta^{i\prime} = \eta(x^i - K^i)$

$$\nabla_A : \begin{cases} \nabla_\alpha = E_\alpha{}^\beta D_\beta + E_\alpha{}^\mu \partial_\mu + \phi_\alpha M \\ \nabla_a = E_a{}^\beta D_\beta + E_a{}^\mu \partial_\mu + \phi_a M \end{cases}$$

$\to \qquad \nabla_A{}' = e^{-K} \nabla_A e^{+K}$

$$[\nabla_A, \nabla_B\} \equiv T_{AB}{}^C \nabla_C + R_{AB} M$$

Constraints : $\qquad T_{\alpha\beta}{}^\gamma = 0, \quad R_{\alpha\beta} = 2(C\gamma^3)_{\alpha\beta} R$

$\qquad\qquad\qquad T_{\alpha\beta}{}^c = i2(\gamma^c)_{\alpha\beta}$

$$\hat{E}_\alpha = D_\alpha + (\gamma_c \gamma^d)_\alpha{}^\beta H_\beta{}^c \partial_d$$

$$\hat{E}_a = i\tfrac{1}{4}(\gamma_a)^{\alpha\beta} [\hat{E}_\alpha, \hat{E}_\beta\}$$

$$\hat{C}^a = \tfrac{1}{2}[(\gamma_b\gamma_c)^{\alpha\beta}(D_\alpha H_\beta{}^b) + (\gamma_b\gamma_d\gamma_c\gamma_e)^{\cdot\alpha\beta} H_\alpha{}^d \partial^b H_\beta{}^e]$$

$$\times \epsilon^{\cdot\cdot} \Delta^{-1} \epsilon_{\cdot},$$

$$\Delta = \delta + i\tfrac{1}{2}(\gamma\gamma\gamma)(DH)$$

$$E_\alpha = \Psi^{-1/2}(\delta - ig(\tau)\hat{C}\gamma_5)\hat{E}_\beta \qquad g = \tau^{-1}[(1+\tau)^{1/2} - 1]$$

H, Ψ : unconstrained $\qquad \tau = (C)^2$

↳ related to $\det g$.

<u>Mansouri</u> For ~~4~~-dim objects : $SO(P,2)$

1) n-pt fns
2) 4-pt construction
3) factorization
4) special.

$$A_N = \int d^4V\, K(x_1^\mu, \dots x_N^\mu) \qquad \text{Irrep } SO(P,2)\ \phi^{d\beta m}(x^\mu, z)$$

P parameters : $z \to 0$

$$K = \Pi[(x_i - x_j)\cdot(x_k - x_l)]^{-d_{ijkl}}$$

= fn of ~~R~~. anharmonic ratios.

$$d^4V = \prod_{i=1}^{n} d^4x_n \left(\Pi (x_i - x_{i+1})^2\right)^{-2} \qquad R_1 = \frac{(x_1-x_2)^2(x_3-x_4)^2}{(x_1-x_3)^2(x_2-x_4)^2}$$

$$A_i = \int \Pi d^4x\, [R_1]^{-d_{12}-2}\, [R_2]^{-d_{23}-2} \qquad R_2 = \frac{(x_1-x_4)^2(x_2-x_3)^2}{(x_1-x_3)^2(x_2-x_4)^2}$$

Fix $x^\mu = I^\mu \quad x_1^\mu = \infty \quad x_3 = 0$

$$A_4 \to \int d^4x\, (x^2)^{-d_{12}-2}\, [(I-x)^2]^{-d_{23}-2}$$

$$A_4 = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))\,\Gamma(1-\alpha(u))}{\Gamma(1+\alpha\ \ )\ \Gamma(1+\alpha\ \ )\,\Gamma(1+\alpha\ \ )}$$

$\alpha_{12} = \alpha(s) - 1$

$\alpha_{23} = \alpha(t) - 1$

$\alpha(s) + \alpha(t) + \alpha(u) = 1$

Factorize: $\exp\left[-2\alpha \sum_{\sigma=1}^{\infty} \frac{ch\,\sigma\gamma}{\sigma} x^\sigma\right]$

$I_1 = (ch\beta,\ sh\rho,\ \sin\theta, \cos\theta_0 \cdots\ )$

$H_0 = -\alpha(s) - c - R, \quad R = \sum n\, a_n^{\mu +} a_{n\,\mu}$

$V = i e^{i\sqrt{2} k_\mu \psi^\mu} \quad \psi = \sum D\, a^\mu r^\sigma + D^+ a^{\mu +} r^{-\sigma}$

Okubo. E_8.

1. Index
2. Anomaly
3. Derivation of Octonions, Jordan etc.

Casimir $I_3 = 0$ if $L \neq SU(n)$, $n > 3$

$I_4 = \cdot\cdot$

Racah

$SU(n)$ $2, 3, \cdots n$

$Sp(2n), SO(2n+1)$: $2, 4, 6, \cdots 2n$

$SO(2n)$: $2, 4, \cdots 2(n-1)$; n

G_2 : $2, 6$

F_4 : $2, 6, 8, 10$

E_6 : $2, \overset{5}{6}, 8, 9, 10, 14, 18$

E_7 : $2, 6, 8, 10, 14, 18, \cdots$

E_8 : $2, 8, 10, 14, \cdots \quad 30$

Index $D_\mu(\rho) \equiv \mathrm{Tr}(\rho)\, I_\rho \qquad D_\sigma(\rho) = 0 \ \forall\ E_\rho$

$\rho_A \otimes \rho_B \quad \Rightarrow \quad d_A d_B = \sum d_j$

also $\quad \cancel{D_A D_B =} \quad d_A D_B + D_A d_B = \sum D$

SU(2) $\quad I = 1 \quad \sum_{\text{cycl}} \xi_1 \times (\xi_2 \times \xi_3) = 0$

$I = 2: \quad 2 \times 2 = 3 + 2 + 1 + 0$

$R * \gamma = \frac{1}{2}\{RS + SR\} - \frac{1}{3}\delta(RS) \quad \to \quad R * S = S * R \quad$ pre-Jordan

Jordan identity $\quad (((x * x) * y) * x) = (x * x) * (y * x)$

Jordan-Wigner-von Neumann Th.

G_2: $\quad \square = 1$-dim

$\boxminus = \oplus \rho_i = \boxminus + \cdots$

$\Rightarrow [x, y] = -[y, x]$

$\boxplus = \square + \cdots$

$F_4 \quad \boxplus = \square + \cdots$

$E_6 \quad$ triple product : Brown-Yang construction

<u>Kawai</u> Polyakov str.
-type

a) random walk. 

$\exp\{-m\sum|x_i - x_{i-1}|\}$

replaced by $\exp\{-\frac{1}{2a^2N}(x_i - x_{i-1})^2\}$

b) locality : $G(xyz) = G(xy)\, G(yz)$

determines the weight.

b) random surfaces

$W(C) = \int \mathcal{D}x\, e^{-\rho_0 \int \sqrt{\frac{\partial(xx)}{\partial(\xi)}}\, d^2\xi} = \int e^{-\rho \sum \text{areas}}$

C

$\partial S = C$.

triangulation.

locality or factorizability

Cont. limit. $S = \int K^{ab} \partial_a X^\mu \partial_b X^\mu d^2\xi + S_0(K)$

$K^{ab} = \underbrace{g^{ab}}_{\det = 1} \sqrt{g}\, \phi$ det

2-dim)

Conformal inv. theories

i) $x \to x + c$ ii) $x \to Rx$ iii) locality OK.

iv) positivity

$c = 26$ $S = s(x^\mu, \phi) + \frac{c}{24}(\partial_a \varphi)^2$ $g_{ab} = e^{-\varphi}\delta_{ab}$

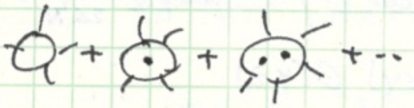
If $c = 26 \Rightarrow$ conformal = general cov.

$$\int dg = \int \mathcal{D}\varphi\, e^{-\frac{26}{24\pi}(\partial\varphi)^2}$$

Compactification and nonlinear σ

$$\int \partial_a X^\mu \partial_a X^\nu g_{\mu\nu} \, d^2\xi$$

metric $\Leftrightarrow$ closed string ?

Dilatons  renormalizes $(\partial X)^2$

gravitons similarly generates $g_{\mu\nu}$ self-consistently ?

3° Finiteness of O(32)

$$A_P = 16\pi^3 g^4 K \int_{0 \le \nu_1 \le \nu_2 \le \nu_3 \le 1} d\nu_1 d\nu_2 d\nu_3 \int_0^1 \pi \frac{dq}{q} \pi (\psi'_{II})^{k_i \cdot k_j}$$

$$A_N = -K \cdots$$

$$\psi_P', \quad \psi_N'$$

<u>Staebler</u> SUSY Discussion

Logic diagrams.

Yukawa coup. depends on the manifold

Proton decay: dim 4 op. unless discrete sym.

CP viol. from manifold. ~~but~~ but

$E_8 \to$ ~~S~~ $SU(5) \times G$ ruled out

RGE $\Rightarrow$ 4 families

$\sqrt{M_W M_P} \sim M_P$

$\sin^2\theta_W$ not very good. $SU(4) \times SU(2) \times SU \times SU$.

SU

3 families better

$SU(3)_L \times SU(2)_L \times$ ~~S~~ $U(1)_Y \times (U(1))^2$ or $SU(2) \times U(1)$)

Chang. finiteness ?

Gates : non-Kähler with torsion.

Marshak preons $SU_c \times SU(N)_L \times SU(N)_R \times U(1)_L \times U(1)_R$

Metcolor group FFF $\in$ singlet $\to$ $SU(3)$, $E6$

$E_6 \Rightarrow E_6 \times SU(16) \to SO(10)$

$\hookrightarrow N < 22$

preons $(27, 16)$ $\to$ analog of $SU(6)$ hadrons

$\to$ $16 + 144 + 1200$ 3 generations

$(4, 2, 1) + (\bar{4}, 1, 2) + \cdots$

masses $\sim$ Casimir $\to$ $4 : 35 : 470$