

# Gross

Anomaly: charm for GIM  
free heterotic

string = massless free field on cylinder  
interaction introduced. 1st quantiz.  
↳ but built in as topology (still orientable) <sup>surfaces</sup>

heterotic:

}	Right	$X^{\mu}, \psi$	$X, X$
	Left	$X, \psi$ $D=10$	$D=24 = 8+16$

$\left( \begin{matrix} X_R \\ X_L \end{matrix} \right) \oplus \left( \begin{matrix} \psi_{802} \\ X_{16} \end{matrix} \right) \rightarrow 24 \text{ dim}$  ) inherently chiral  
 $D=10$

16 K-K Instantons  
480 massless solitons

$$S = \int d^2\zeta e [ (\partial\alpha X) + \frac{1}{2} \psi^{\mu} \rho \partial\psi + \frac{1}{2} (\partial\alpha X)^2 + \frac{\lambda^{\alpha}}{2} (\rho\rho\rho\psi) \partial X + \lambda (\partial\alpha X)^2 ]$$

↳ Last. multiplied for left mov. only

Bosons  $\rightarrow X_{LR}^{\mu} + X_L^{\text{internal}} + \text{fermi}$   
 $10 \quad 16$   
↳ modified canon. comm.

New contr. on  $\sigma$  &  $p$

$$\frac{1}{4} M^2 = N + \tilde{N} - 1 + \frac{1}{2} (p^+)^2$$

$$N = \sum (\alpha\alpha + \frac{1}{2} \tilde{S} \tilde{S}) \quad \tilde{N} = \sum (\alpha\alpha + \alpha' \alpha')$$

$$X^{\pm} \in T_{16} = \square = R/\Gamma_{16}$$

$$\text{string } S^2 \rightarrow T_{16} \rightarrow \pi_1(T_{16}) = \mathbb{Z}_{16}$$

$$X^{\pm}(t+\sigma) = X^{\pm} + p^{\pm}(t+\sigma) + L^{\pm} \sigma + \dots$$

$$p^{\pm} = \frac{1}{\sqrt{2}} \sum \frac{m_i}{R_i} e_i^{\pm} \quad X^{\pm} = L^{\pm} \sigma \pm \sqrt{2} \sum \dots$$

integer, even, selfdual  $\rightarrow$  interacting th.  
 $(p^+)^2 = 2, 4, \dots$

self-d:  $d=8, 16, 24$   
 $\Gamma_8, \Gamma_{8+8}, \dots$

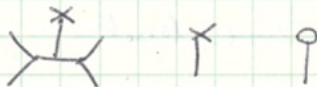
$E_8 \times E_8$



$\Gamma_{16}$

~~$\Gamma_{20}$~~   $\Gamma_{20}$

Proof of finiteness:



Conclusions

Disc.  $E_8$  heterotic str  $\rightarrow$  Kac-Moody sym  
 Type I not

T. Banks gauge inv & coord inv. (Kaku, Siegel)

$$\Phi[X_{\mu}^a] \sim A_{\mu}(x)$$

projection 1. and long

$$L_n = : \sum_m \alpha_m \alpha_{n-m}$$

$$L_{-n} \quad n > 0 \quad \text{gauges.} \quad \delta\phi = \sum_{\oplus} L_{-n}$$

$$\langle \phi | P(L_0 - 1) P | \Phi \rangle \quad L_0 | \phi \rangle = 0$$

Lowest:  $|0, p\rangle$

$$L_0 = p^2/2 + N$$

$$\int d^D p \varphi(p) \varphi(p) \quad (p^2 = 1)$$

next

$$\int d^D p A_{\mu}(p) \alpha^{\mu} \Rightarrow \text{transverse projection}$$

Reproduces Maxwell action.

2nd level:  $L_n^u, \tilde{L}_n^u$   $\alpha_n^u, \tilde{\alpha}_n^u$   $(L_0 = \tilde{L}_0 = 0 \text{ constraint})$   
 $\langle \Phi | P (L_0 + \tilde{L}_0 - 2) P | \Phi \rangle$

First excited states

$T^{\mu\nu} = \pi^{\mu\nu} \alpha D(p)$   $\rightarrow$  dilatation

$\hookrightarrow$  projector  $\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$

$\rightarrow \delta T_{\mu\nu} = p_\mu \Lambda_\nu + p_\nu \Lambda_\mu$  linearized gen. coord.

$t^{\mu\nu} \sim \alpha_n^u \alpha_{-n}^{\tilde{\nu}} |0, p\rangle$   
 $= S + A$   
 $\hookrightarrow$  gauge field action

More gauge inv. on 2nd level

$(T_m(p) - \alpha_n^m - \alpha_n^{\tilde{m}}) : G^{\mu\nu}, \pi \pi - \frac{1}{\alpha} \pi \cdot \pi$

2nd  $\delta T^{\mu\nu} = \frac{1}{4} \eta_{\mu\nu} \Lambda$  linearized Weyl

$\delta T_m = \partial_m \Lambda$

$\delta \phi = A$

Each level has a rich gauge structure (in  $D=26$ )  
 but nonlocal effective  $L$ .

Reproduce Liéger

$\sim$  Lorentz gauge  $\rightarrow L_0 = -1$

Faddeu-Pop.  $\sum_{m,n>0} \langle C_m L_m L_{-n} C_n \rangle$

$\sim (\partial_\mu V^\mu)^2$

$\Psi_{\mu\nu} = -\Psi_{\nu\mu}$

inv under  $\delta |C_n\rangle = \sum L_m | \psi_{0m} \rangle + \sum C_{nmk} | \psi_{nk} \rangle$

$\Rightarrow$  New Ghosts.  $C_{\mu\nu}$  etc. ...

$\hat{\Phi} [x_\mu, \theta, \hat{\theta}] \rightarrow$  expansion in  $\theta, \hat{\theta}(\tau)$

# E. Witten Global Grav Anomalies

$$\delta \mathcal{L}_{\text{eff}} \rightarrow \int D_n(L)^\mu \neq 0$$

Global:  $S^d \rightarrow G \quad \pi_d(G) \neq 0$

- 3 reps.
1. real  $\det \phi > 0$  or
  2. pseudoreal  $\det$  not real, not definite  $\rightarrow$  ~~±~~ sign anomaly
  3. complex  $\det \phi = \text{complex}$
- even if L.A. = 0, may be  $\exists$  G.A.

Grav. anom.  $D_n \langle T^{\mu\nu} \rangle \stackrel{?}{=} 0$  Local

Disconn. orient conv. differ  $\# S^{10}$  has 992 components

Indeed  $S^d$  in  $d+1$ -manif.

$$S^n: S^{n+1} \xrightarrow{x} S^n \quad x \rightarrow g(x)S$$

$S^{n+1} \rightarrow S_g^{n+1}$  If  $S_g$  homeomorph (not diffeo): exotic sphere  $S^{n+1} \iff$  diffeo classes of  $S^n$

Next  $S_g^{n+1} = \partial B$

$B: d=12 \quad P_1^3 = \int (Tr R^2)^3 \quad P_1 P_2 = \dots$  ) top inv.

$$d Tr R^2 = 0 \not\Rightarrow Tr R^2 = dH$$

But it may be so on  $\partial B$ . (sugy..)

def:  $P_1^3 = \int (Tr R^2)^3 - \int H (Tr R^2)^2$ , etc

But no analog of  $P_3$  exists

$$\det \phi = \text{integer} = \text{fn}(\text{top inv.})$$

Milnor's work on  $S^{11} \rightarrow 992$

Generalization 10 dim spin man.



cylinder  $Q \times I$   $I = [0, 1]$

$$(Q \times I)_g : (x, 0) = (g(x), 1)$$

Consider  $L_{\text{eff}} = \ln \det i\cancel{\not{D}} + \text{counterterms}$   
 $h \rightarrow h^t$

$$\delta L_{\text{Dirac}} = \text{Tr} \frac{1}{i\cancel{\not{D}}} \gamma^\mu \delta A_\mu$$

Choose  $h_{\mu\nu}^t = h_{\mu\nu}^0 + \delta h_{\mu\nu}^1(t)$

$$\Delta = \int_0^1 \frac{d}{dt} L_{\text{eff}}(h^t) \quad \text{compute in terms of top. inv.}$$

Atiyah-Singer :  $\eta(M) = \lim_{s \rightarrow 0} \sum (\text{sign } \lambda_i) \cdot (\lambda_i)^{-s} \quad \cancel{\not{D}} \text{ on } M.$

$$\rightarrow \eta = \text{index}(\cancel{\not{D}})_B - \int \hat{A} \quad \hat{A} : \text{curvature polyn. of } M = \partial B$$

$$\Delta \Rightarrow \eta(Q \times S^1)_g$$

Let  $L_{\text{eff}} = \frac{1}{2} \ln \det i\cancel{\not{D}} \left( \frac{\cancel{\not{D}}}{2} \right) \quad \Gamma = \Gamma^1 - \Gamma^{(10)}$

$$\rightarrow \frac{dL_{\text{eff}}}{dt} = \frac{i}{4} \sum \frac{1}{\lambda_i} \langle \psi_i | i\cancel{\not{D}} | \psi \rangle$$

$$i\cancel{\not{D}}'' : \left( i\bar{\Gamma} \frac{d}{dt} + iD^t \right) \Psi = \lambda \bar{\Psi} : \text{adiabatic sol.}$$

$$\Psi = A(t) \psi_0^t + B \bar{\Gamma} \psi_0^t$$

$$U \begin{pmatrix} A \\ B \end{pmatrix} = \lambda \begin{pmatrix} A \\ B \end{pmatrix} \quad U = i\sigma_1 \frac{d}{dt} (\mu_0(t) + \sigma_3 \lambda_i)$$

$$\mu = \langle | \bar{\Gamma} | \rangle$$

$$\lambda_n^\pm = \mu_0 \pm \sqrt{(2\pi n)^2 + \lambda_0^2} \rightarrow \eta_\mu = \mu_0$$

$= \int_0^1 dt \mu_0(t)$  in general

$$\eta = \int dt \sum \frac{1}{\lambda_i - \lambda_j} \langle \psi_i | \Gamma | \psi_j \rangle \langle \psi_j | \frac{d}{dt} i\cancel{\not{D}} | \psi_i \rangle$$

$$\rightarrow \sum \frac{1}{2\lambda_i} \langle \frac{d}{dt} i\cancel{\not{D}} | \psi \rangle \Gamma$$

$$\mathbb{I}B \quad \Delta S_{\text{eff}} = 2\pi i \sigma(B)/\rho$$

$O(3,2)$ ,  $E_9 \times E_9$ : ..

$$\text{Milnor: } (S^{10} \times S^1)_g \rightarrow \partial B, \quad P_1 = P_2 = 0 \quad \sigma(B) = \mathbb{R}Z$$

$$\rightarrow \Delta S_{\text{eff}} = 0 \pmod{2\pi i} \rightarrow e^{i S_{\text{eff}}} = \text{single valued.}$$

String th

i) one-param fam of metrics on  $\Sigma$  (mod. transf)

ii) " mapp  $\Sigma \rightarrow \cdot$

$$\Sigma \times S^1 \rightarrow Q \text{ (sp-time)}$$

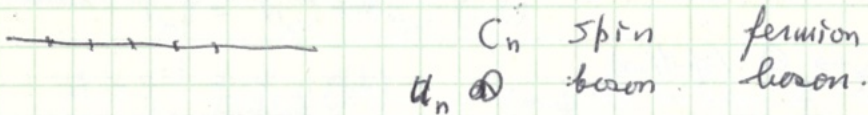
$P_1, C$

Case of torsion:

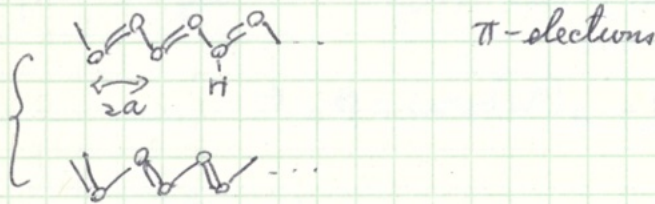
# Schnieffer

Frac charge & topol ~~in~~ kinks

1. Dyn. sym br. in coupled ferm-bos sys in (1+1) Piers Thm
2. stable exc. are top. kinks
3. sharp frac. ch.
4. frac.  $\sigma$ -Hall eff.
5.  ${}^3\text{He}$  in A.



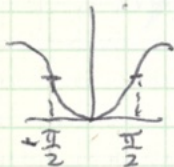
Piers Thm.  $\langle u \rangle \neq 0$



Su, Heeger, ... kinks width  $\sim 14a$   
mass  $\sim 6m_e$

$\phi_n \equiv (-1)^n u_n$  staggered field

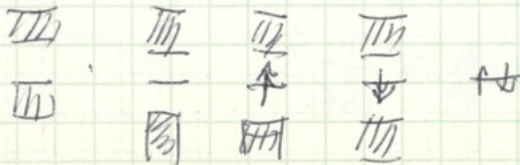
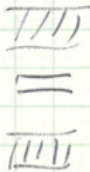
$\Rightarrow$  Bloch states.



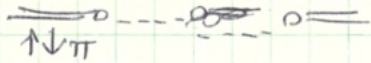
$\sim$  spin  $\frac{1}{2}$  fermion

$$H_{\text{cont}} = \int V_F \psi \sigma_i \frac{\partial}{\partial t} \psi + g \psi_c^\dagger \phi \psi$$

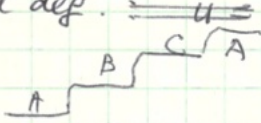
(Tachik & Rebbi)



TTF-TBQ at 1 kbar  $\neq$  2 e/3 atoms



3 fold def.



displaces 2 electrons among 3 kinks



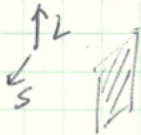
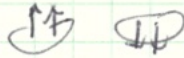
Sharpness?

Bell-Fate Rajamou  
Tachinun

$$\delta Q^2 = 0 \quad (L \rightarrow \infty)$$

Q. Hall: helical junction

Super  $^3\text{He}$



spin current.

# Zumino Coho & anomalies

Simplicial homoge. group coho.

$g_0, \dots, g_n \in G, \rightarrow f(g_i)$  cochain

$$\delta f(g_0) = f(g_1) - f(g_0) \equiv \delta f(g_0, g_1)$$

$$\delta f(g_0, g_1) = f(g_2) - f(g_0, g_2) + f(g_1, g_2) \equiv \delta f(g_0, g_1, g_2)$$

$$\delta^2 = 0. \quad \text{normalized so } f = 0 \text{ if any pair } g_i = 0$$

$f$  may be in the rep space of  $G$ .

$$f(A; g_0, \dots, g_n) \quad A \rightarrow Ag \quad \text{rep space.}$$

$$\Rightarrow \text{inv: } g f \equiv f(A, g^{-1} g_0, \dots) = f(A, g_0, \dots) = \delta(g f) = g(\delta f)$$

inhomo gr coho.

$$g_0 = \sigma_0 \quad g_1 = \sigma_0 \sigma_1, \dots \quad g_n = \sigma_0 \dots \sigma_n$$

$$f(A, g_i) = \hat{f}(A; \sigma_i)$$

Let  $f$  be inv.

$$\text{Def: } \varphi(A; \sigma_i) \equiv \hat{f}(A; 1, \sigma_1, \dots, \sigma_n) \quad : \text{inhomo cocyc.}$$

$$\delta \varphi(A) \equiv \varphi(A\sigma) - \varphi(A) \equiv (\delta \varphi)(A; \sigma) \quad \text{etc.}$$

$$\delta^2 = 0$$

$$\delta \varphi(A; \sigma_1) = \varphi(A\sigma_1, \sigma_2) - \varphi(A; \sigma_1, \sigma_2) + \varphi(A; \sigma_1)$$

$$\equiv (\delta \varphi)(A; \sigma_1, \sigma_2)$$

(Faddeev cocycl.)

$\rightarrow$  proj. rep. of  $G$

(Baymoun & Wigner)

$$U(A; \sigma)$$

$$U(A; \sigma_1, \sigma_2) = U(A; \sigma_1) U(A\sigma_1; \sigma_2) \times e^{-i\varphi(A; \sigma_1, \sigma_2)}$$

$$\text{Associativity: } \text{const. } U(A; \sigma_1, (\sigma_2 \sigma_3)) = U(A; (\sigma_1, \sigma_2) \sigma_3)$$

$$\Rightarrow \delta \varphi = 0$$

(Frobenius)

$\Rightarrow$  anomalous Schwinger terms in Lie alg.

Lie alg.: diff form in group sp.

$$\delta \text{ ext. diff} \quad v = g^{-1} \delta g$$

$$\delta v = -v^2$$

$$\text{Let } A_g = Ag \Rightarrow \delta A_g = A \delta g = A_g v$$

$$\left\{ \begin{array}{l} \delta v = -\delta^2 \\ \delta A_g = A_g v \end{array} \right. \quad \text{BRST}$$

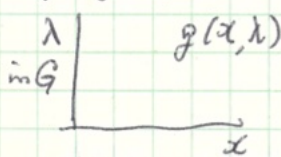
Let  $P$  a form of deg  $n-1$

$$0 = \int_{V_n} \delta P = \int_{\partial V_n} P \quad \text{exp. } V_2 = \begin{array}{c} g_1 \quad g_2 \\ \diagdown \quad / \\ g_0 \end{array}$$

$$\int_{\partial V_2} = \int_{g_0}^{g_1} + \int_{g_1}^{g_2} + \int_{g_2}^{g_0} = \quad (\text{hom. cocycl.})$$

If  $\delta P = 0 \rightarrow$  for small path  $\delta$  change,  $\delta^2 = 0$

Gauge gr. case.



$$d = dx \frac{\partial}{\partial x^\mu}$$

Local coho.

$$\delta = d\lambda \frac{\partial}{\partial \lambda^i}$$

(not well defined)

$$A_\theta = A_\mu(x) dx^\mu$$

$$A_g \equiv g^{-1} A g + g^{-1} dg$$

$$v = g^{-1} \delta g$$

Find  $P(v, A_g) : \delta P = 0$  (polym.)

descent eqs

$$F = dA + A^2, \quad F_g = dA_g + A_g^2 = g^{-1} F g$$

$$\omega_{g_{n-1}} = n \int dt P(A_g, F, \dots), \quad P = d\omega_{2n-1}, \quad \delta P = 0$$

$$\delta \omega_{2n-1} = -d\omega_{2n-2}^1$$

$$\delta \omega_{2n-2}^1 = -d\omega_{2n-3}^2$$

Anom. sequences

$$\delta \omega_0^{2n-1} = 0$$

(Russian formula)

Integrating over  $x \rightarrow \int_x \omega_{2n-1-k}^k = \alpha_k, \quad \delta \alpha_k = 0$  closed.

intef over  $\lambda$

$$\int$$

$\rightarrow$  Schwinger form.

Recent develop

1. improve & streamline
2. minimal form of the  $\omega$
3. background fields.

generation of gauge-transf.

$$G^a = \partial_i E^{ia} + f^{abc} A^b E^c$$

$$G_i^a = \hat{G}_i^a - \psi \delta^{ota} \psi$$

Calc. ETCom. by BJL limit

$$[G^a(x), G^b(y)] = i f^{abc} G^c(x) \delta^3(x-y)$$

$$+ \frac{i}{24\pi^2} d^{abc} \epsilon_{ijk} \partial_j A_i^c \partial_k \delta^3(x-y)$$

$G^a \Psi = 0$  not consistent. (obstruction to quantization)

Kobayashi + Sugawara

$$\hat{G} = \hat{G} - \psi \delta^{ota} \psi \rightarrow G^a \Psi = 0$$

Susy anomaly. (rigid)

$A, \lambda, D$ .

perturb. cal.

coho.

Nielsen; Gaadagnini, Konishi, Mutsaers

Piguet Sibold

Sonora et al

Cicardi et al

Anomalies in SUSY

Einstein

Local Lorentz

Local SUSY

↕ → when it happens → no susy anom. too  
Baulieu

# ~~M. Bowick~~ ~~Susp lat string~~

## Briick Uniquens of Susp action

Functional basis  $\leftrightarrow$  oscillator basis

2. Field in light cone gauge & light cone frame.

$$x^+, \bar{p}^- = H, \quad A^+ = 0 \rightarrow \text{solve for } A^-$$

$A^\pm$  (transv.) - dynamical

$$L = \frac{1}{2} \bar{A}^2 \square A^2 + g^{\mu\nu} A^\mu A^\nu + g^{\pm I} A^\pm$$

nonlinear Poincaré:  $J^{\pm-}, J^{\pm i}, P^- = H$

Susy  $\phi(x^\mu, \theta^m, \bar{\theta}_m)$   $Q \rightarrow \begin{matrix} Q_+ \text{ linear} \\ Q_- \text{ nonl.} \end{matrix}$

$$\{Q_-, Q_+\} \rightarrow H$$

$d=10$  suYM  $\phi(x, \theta_A)$

$\rightarrow \mathbb{R}^4$  under  $SO(4) \times SO(10)$

$\mathbb{I} \phi_a$  sugy  $\psi(x^\mu, \theta^A)$

$\rightarrow \mathbb{R}^8$  of  $SO(8)$

$\mathbb{I} b$  str.  $\bar{\Psi}[x^\mu(\sigma), \theta(\sigma)]$

$$\rightarrow \mathcal{L} = \bar{\Psi} \square \Psi + W " \delta^2 \Psi^3 "$$

3. Kinetic counterterms

$$\mathcal{L} = \bar{\phi} \square \phi + \frac{\alpha}{2} \bar{\phi} \square^2 \phi ?$$

Unique if local terms only

$\alpha = 0 \rightarrow \pi \nu \partial \phi$  primary constraint

$\alpha \neq 0 \rightarrow$  first order form.

$$\rightarrow \mathcal{L} = \frac{1}{2} (\bar{\Psi}_1 \square \Psi_1 - \Psi_2 \square \bar{\Psi}_2 + \frac{1}{\alpha} \bar{\Psi}_1 \Psi_2)$$

ghost

Check Poincaré inv.

IIb sugy with counter terms: OK too.  $SU$

Sust. IIb  $\Sigma = (X^T X^T, X^i(\sigma), \theta^a(\sigma))$

$\bar{p}_i = \delta / \delta x^i, \bar{d}^a = \delta / \delta \theta^a(\sigma)$

$P^- = h = \frac{i}{2\pi} \int d\sigma \{ \delta^i \delta^j + x^i x^j + \dots \}$

good only for  $d=10$ .

Q: In general, add counter terms?

$\vec{p}_{1, \text{new}}^a = M \tilde{\theta}^a + F \tilde{\alpha}(\tilde{\theta}, \tilde{d})$

$\vec{p}_{2, \text{new}} = \frac{-M}{2} \tilde{d}^a - \frac{1}{2} G^a(\tilde{\theta}, \tilde{d})$

Close the alg on  $\mathbb{R}V_3$

Higher  $V$ 's not allowed.

Q. Can change the str dim from 10 (26) to any by similar way  
Gervais: Yes.

J. Harvey Heterotic str.

1. Fermionic formulation

2. Interactions

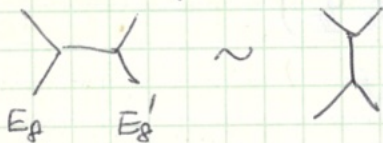
2 fermions for each ~~deg~~ boson

1. 16 bosons  $\rightarrow \psi^I, \bar{\psi}^I$   $I=1,2,\dots$   $O(32)$   
 $U_p \times U_p$

2. Virasoro

~~$F(b-1-s)$~~

Shapiro-Lipshutz amplit.



# Siegel cov. formal.

Sometimes cov. gauge lattice than light cone.

Anomaly in str. : open str  $\rightarrow$  closed  $\rightarrow$  cov. dev. in BRST: for str.

1). Free  $G_i \rightarrow$  free  $Q_0 - L_0 = \frac{1}{2} \Phi K_0 \bar{\Phi}, [Q_0, K] = 0$

(2)  $Q = Q_0 + g Q_1 + \dots, Q^2 = 0 \rightarrow Q_0^2 = 0, [Q_0, Q_1] = 0 \dots$   
 $\rightarrow Q_1 \rightarrow L_1$  (Vertex op.)

Bosonic str.  $\beta(\sigma) = \frac{1}{2} (i \frac{\partial}{\partial X} \mp X'(\sigma)) \quad \tilde{C} = \frac{1}{\sqrt{2}} (C \mp \frac{1}{\partial C})$

$G(\sigma) = \frac{1}{2} \hat{P}^2 \rightarrow Q_0 = \hat{C} (\frac{1}{2} \hat{P}^2 + i \tilde{C} \frac{\partial}{\partial C})$  on  $\Phi(X, C, \bar{C})$

$K = \frac{\partial}{\partial Q} \rightarrow [Q, K] = 0$

when open:  $Q = i \frac{\partial^2}{\partial \sigma^2} \rightarrow K_0 = c H = \frac{\partial}{\partial C} \hat{H}$

closed  $Q = \frac{1}{2} [C \frac{\partial^2}{\partial C^2}] \rightarrow K_0 = c (c H - \frac{\partial}{\partial C} \hat{H}) - \frac{1}{2} [C \frac{\partial^2}{\partial C^2}] (N_L - N_R)$

$H = (\frac{1}{2} \hat{P}^2 + i \tilde{C} \frac{\partial}{\partial C}) \neq \rightarrow C = \int d\sigma C, \bar{C} = \int d\sigma \bar{C}$

- $\Rightarrow$
1. Stueckelberg form of massive vectors
  2. Faddeev-Popov
  3. Not all modes appear: at  $C \neq \bar{C} = 0$ .

4. What is gauge inv?

$\delta \bar{\Phi} = \int X' \frac{\delta}{\delta X} \Lambda_r [X]$

Sust. proper time  $\tau$  for particles  $X(\tau), \theta(\tau)$   
 $\delta \theta = c \delta X = c c \gamma \theta$

$L = \int e^{\Lambda} (z) p_A + \dot{y} \cdot f(p) = (\dot{X} + c \dot{\theta} \gamma) p + [-\frac{1}{2} p^2 + \frac{1}{2} \gamma \pi]$

$(g, \psi) =$  gauge fields & Lap. multipl.

$(p, \pi) =$  <sup>conj.</sup> momenta

BRST generators  $G_i = p^2, \psi \pi, \psi \pi, \bar{\psi} \pi = -2\psi, Q = \psi^2 - \psi \pi + (c \psi \gamma)^2 / 2c$

Conformal sym + susy algebr:

$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + \gamma \beta^\alpha + \frac{1}{2} \gamma \gamma^\alpha \partial \theta \theta$

$\gamma = 3$ -indices

$P = \hat{P} + i \gamma_\alpha \partial^\alpha$

$\Omega = \textcircled{H}$

M. Kaku Cov. quantiz.

1st quant. → interaction by hand

2nd " → ~~no~~ no double count.

how to incorporate Virasoro.

$$L_0 = i \bar{\Phi}^\dagger \hat{H} \Phi - \Phi^\dagger (L_0 - 1) \Phi \quad L_0: \text{transverse modes only}$$

Instead:  $\sum c_n L_n \bar{\Phi}(x)$

$$\rightarrow \frac{H_n}{L_n} = \int d\tau d\sigma e^{i p \cdot x} = \int d\Omega \bar{\Phi} e^{i \int \bar{\Phi}^\dagger H \Phi} \bar{\Phi}(x_1) \bar{\Phi}(x_2) \Delta$$

$$\left( F_{\mu\nu} \sim A_\mu P_{\nu\lambda} A_\nu \right) \xrightarrow{\text{proj.}}$$

$$= \int d\Omega x d\Omega p d\Omega \lambda d\Omega p e^{i \int d\tau d\sigma (\rho \dot{x} + \lambda (\dot{p}^2 - x^2) + \mu \rho \cdot x)}$$

$$\text{IIIIII} : e^{i p \cdot x} \Rightarrow e^{i p_i (x_i - x_{i+1})} \rightarrow \langle x_i | p_i \rangle \langle 1 | \dots$$

$$\Rightarrow \langle x | p \rangle e^{i \int (\lambda \dot{x} + \mu \dot{p})} \int d\rho_i d\rho_i'$$

↓  
δ. fno

$$\bar{\Phi}(x) = |x\rangle \equiv \prod |k_n\rangle = \exp(\dots) |0\rangle$$

$$\delta \langle \phi | x \rangle = \epsilon_n \int (-\partial_x^2 + x^2 + 2i x' \partial_n) e^{i n \sigma} \delta \tau \langle \phi | x \rangle$$

$$\rightarrow \delta \phi = \epsilon_n L_n(\partial x, x) \phi(x)$$

$$\mathcal{L} \sim \rho \dot{g} + \lambda n C_n \quad \delta \lambda_n = \dot{\epsilon} + f_{mnp} \rho \lambda_m \epsilon_n$$

$$\delta \lambda = \dot{\epsilon} - g' \quad \delta \rho = \dot{g} + e' \quad \text{ghosts}$$

gauge invariance, Lorentz inv. action → B obtained.

↓  $H(x|y)$  postulate as

$$\mathcal{L} \sim -\frac{1}{4} F_{\mu\nu}^2 + \rightarrow \text{gravity linearized}$$

Brower-Thorn off shell version

Luistr. many constraints = 1st class & 2nd class.

$$\theta\theta = 0 \xrightarrow{\text{but}} \tilde{\theta}\gamma^{\mu}\theta \neq 0$$

Cov. case. change to  $\theta\pi\theta$

by  $P_a = 1 + \frac{\pi\pi}{2\pi^2} \pi$

Also. Hori & Kimura

### M. Bowick High T

$$\rho(n) = n^{-a} \exp(bn)$$

Typ I  $a = 9/2$   $b = \pi\sqrt{2}$

Typ II  $a = 10$   $b = \pi\sqrt{2}$

Het.  $a = 10$   $b = (2+\sqrt{2})\pi\sqrt{2}$

$$\text{B. } \ln Z \rightarrow \left(\frac{T_0}{T-T_0}\right)^{1/2-a} \Gamma\left(\frac{1}{2}-a, \frac{f(T_0-T)}{T \times T_0}\right)$$

Typ I

Typ II, het.

$p, c_v, \epsilon \rightarrow \infty$   
 $\omega T \rightarrow T_0$

$p, \epsilon < \infty$   
 $c_v$

Fluctuations: recipe for hetero. too (Frantchi Carlotz)

$$S \sim a \ln E_s + b E_s$$

$$T = E_s / (b E_s - a)$$

$T > T_c$ ? equib bet massive & massless?

$$bT_{\min} = \frac{20bE - 9a \pm \sqrt{81a^2 + 40iabE}}{20(bE - a)}$$

$$\rightarrow E_p < F - \frac{aT_{\max}}{bT_{\max} - 1} \quad \checkmark < E_n$$

# Gervais

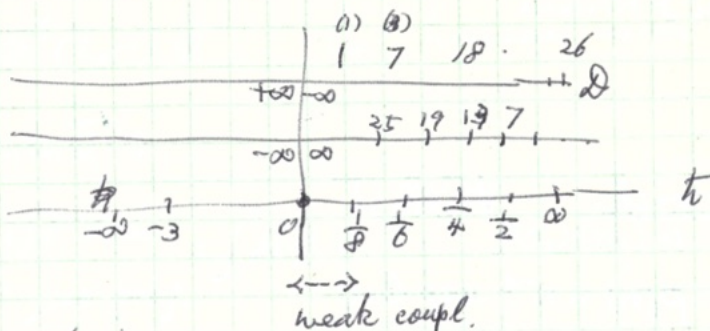
Liouville  $S = \frac{1}{16\pi\alpha'} \int [ \frac{\dot{\phi}^2}{2} - \frac{q_0^2}{2} - e^\phi ] d\tau d\sigma$

$\hbar$  param.

$L_n; C = 1 + 3/\hbar$

$X_0 \Rightarrow$  ghost

$\hbar = \frac{N}{2(N+1)^2}$



no tachyon.

zero Liouville state.

Locality OK for

25, 19, 13, 7, 1, 0

3, 5, 7

Strong coupl  $D = 7, 13, 19$

Liouville on faces down