S

J. Schwarz.

Type II : only cubic interaction $\overset{\sim}{\Rightarrow}$ N = 8 supergravity

Type I : Q = M/W. chiral = self conjugate
$\Rightarrow$ N = 4 susy.

Type II_A : Majorana, nonchiral

Type II_B : Weyl

10-dim particle th.

1). ~ Type I. susy Y-M.

2). ~ Type I closed. str. sugy N = 1, D = 10

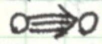
Type II_A $\leftarrow$ ~~d~~ sugy $\leftarrow$ 11 d sugy. nonchiral

Type II_B sugy. $\not\leftarrow$ 11 d.

Contains O(2) acting on N = 2, $\subset U(1,1)$

General : maximal sugy in D $\rightarrow$ $E_{11-D,\,11-D}$

Construction of root vectors

G_2 e_1 e_2   fundamental rep 

$\sqrt{3} : 1$

$e_1\ (\sqrt{3}, 0)$ $e_2 (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\qquad G_2 \supset SU_3\ (\bullet)$ $\qquad 3 + \bar{3} + 1 \quad SU_3$

F_4 $\quad e_4 \ e_3 \ e_2 \ e_1$  $\sqrt{2}\ \sqrt{2}\ 1\ 1$

$e_1 \quad (1, 0, 0, 0)$

$e_2 \quad (-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0)$

$e_3 \quad (0, -\frac{2}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, 0)$ $\qquad e_1 + 2e_2 + e_3 = (0, \sqrt{\frac{1}{3}}, \sqrt{\frac{2}{3}}, 0) \sim \bar{e}_3\ (SU_5)$

$e_4 \quad (0, 0, -\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}})$

SU_5 

$e_1 \quad (1, 0, 0, 0)$ $\qquad e_1 + 2e_2 + e_3 = (0, \frac{2}{\sqrt{3}}, \sqrt{\frac{2}{3}}, 0) \sim \bar{e}_3\ (F_4)$

$e_2 \quad (-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0, 0)$

$e_3 \quad (0, -\sqrt{\frac{1}{3}}, \sqrt{\frac{2}{3}}, 0)$

$e_4 \quad (0, 0, -\sqrt{\frac{3}{8}}, \sqrt{\frac{5}{8}})$

$F_4 \supset SU(3) \times SU(3)$

$\quad \supset SO(8) \times G_2$

$\quad \supset C_3 \times SU(2)$ $\qquad C_3 = Sp(3)$

Magic square

$C' \backslash C$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{Q}$	Ω
$\mathbb{R}$	SO_3 3 5	SU_3 8 $\bar{6}$	Sp_6 21 14	F_4 52
$\mathbb{C}$	SU_3 8 8	$SU_3 \times SU_3$ 8,1+1,8 $(3,\bar{3})$	SU_6 35 20	E_6 78
$\mathbb{Q}$	Sp_6 21 14	SU_6 35 $\overline{15}$	SO_{12} 66 32	E_7 133
Ω	F_4 52 26	E_6 78 $\overline{27}$	E_7 133 56	E_8 248
	$\times G_2$	$\times SU_3$	$\times SU_2$	$\times 1$

--- $\mathrm{Aut}(\Omega / X)$ $X = \mathbb{R}, \mathbb{C}, \mathbb{Q}$

Algebras ↔ $\mathcal{L}(C, C')$ constructed with Hurwitz algebras C, C' as elements of 3×3 matrices.

Notation: ex: $\Omega/\mathbb{Q}: \underbrace{Sp_6}_{F_4 \supset} \times SU_2$

Red: dim. of ~~nonadj.~~ small rep

Anomaly Symp.

Alvarez: Anomaly & cohom. Charge quant.

1. Chech Cohomo
2. Dirac
3. N.Y.M. Jackive
4. W. Zum.

Local cohom. how to clarify ω (wave fn QM)

Common features:

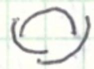
$$\mathcal{L} = \mathcal{L}_0 + T \rightarrow \text{topol.}$$

$$\int A \cdot dx \qquad A \times dA \ldots$$

$$\mathrm{Tr}\, \pi\, d\pi \wedge d\pi \qquad \text{W. Z.}$$

Dirac.  $R \to S^2$  Wu-Yang prescription.

 $d \Sigma f_{ik} = 0$

Cohom.  

0-cochain	$T_{\alpha\beta\mu}$	A_μ
1- "	$J\varphi_{\alpha\beta}$ (trans. f.)	ψ_u
2- "	$K_{\alpha\beta\gamma}$	

Cobound. op. $\delta\{T_\alpha\} = T_\alpha - T_\beta$ etc

Chech cobox

$$\begin{array}{c|cccc}
\Omega^3 & 0 & & & \\
\Omega^2 & dA & 0 & & \\
\Omega^1 & \{A\} & \delta A - d\psi & 0 & \\
\Omega^0 & & \psi & \delta\psi & 0 \\
\hline
 & C^0 & C^1 & C^2 & C^3
\end{array}$$

$\uparrow d \downarrow$ $\qquad \rightarrow \delta$

$\delta C \psi = 0$ → chech 2nd cohom class H

$[d, \delta] = 0$

2-dim. field th. 3rd cohom.

$$\begin{array}{c|ccccc}
\Omega^4 g & 0 & & & & \\
\Omega^3 g & dT & 0 & & & \\
2 & T & \delta T + dJ & & & \\
1 & & J & \delta J & 0 & \\
0 & & & K & \delta K & 0 \\
\hline
 & C^0 & C^1 & C^2 & C^3 & C^4
\end{array}$$

$\rightarrow K_{\alpha\beta\gamma}$

Wu-Yang amb.

$\to \check{K}_{\alpha\beta\gamma}$

$\shortparallel$

$\delta L_{\alpha\beta\gamma\delta}$

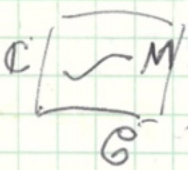
If $\check{H}(M)=0$ no quant.

If $\qquad = R\oplus R\oplus \cdots$

Classifi. wave fu. $\quad d = d_s + 1$ ↳ space

$\varphi \to M \qquad \mathcal{C} = \{\varphi:\}$

$\Psi[\varphi]$



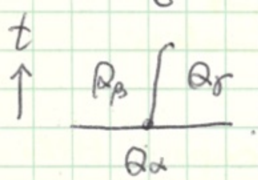
line bundle.

$e^{iS}\Psi = \Sigma e^{iS_i} + \cdots$

$\xi_{12} + \xi_{\cdot\cdot} + \cdots \quad = 0.$

$d = 1+1.$

$e^{iST}\ e^{-i\int_x J_{\alpha\beta}}\ e^{iST_\beta}$



$H^{d_s+2}(M)$

Faddeev. $\quad 4 = 3+1$

$H^2(\ _3) \to H^6(S)$

? What is the "vector potential"

$$\begin{array}{c} SU(3) \\ \Big\downarrow SU_2 \\ S^5 \end{array}$$

$\pi_5(S^5) \to \pi_5(SU_3) \to \pi_5(SU_2) \to 0$

$\qquad Z \qquad\qquad Z \qquad \to Z_2$

$\qquad 1 \quad \longrightarrow \quad 2$

Bott period.

Atiyah Topol. aspects of anomaly

math $\leftrightarrows$ phy

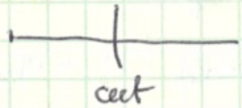
homotopy cohomology dual relation

topol $\xleftrightarrow{\delta}$

diff geom. $\xrightarrow{d}$ Analysis

Laplace op. Δ. eigenvalues > 0

Dirac op. D_t $\quad R$

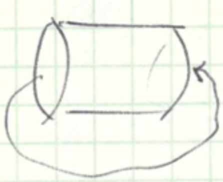


$0 \le t \le 1$

# of crossing $\Rightarrow$ spect flow

Ex. 1. $D_t = i\frac{d}{dt} + t$ $\qquad D_0 \cong D_t$ gauge eq.

2 $\mathcal{D} = D_t + \frac{\partial}{\partial t}$ on T^2 $\qquad \mathcal{D}$ not self-adj.



Index $\mathcal{D}_t$ = dim $\mathcal{D}$ − dim $\mathcal{D}^*$ = 1.

# of zeros eigenvalues

More general. map $D \to D/(D^*D)^{1/2}$ compact Hilb sp.

$\mathcal{S}$ $\qquad Fu = 0$, ~~$F^*v = 0$~~ finite # of zeros

discrete + finite

$\to$ homotopy π_1

more parameters : $\pi_i(\mathcal{F}^1) = \mathbb{Z}$ i odd

$= 0$ even

(Faddeev, 3-d gauge th)

$\pi_i(U(N)) = \begin{cases} \mathbb{Z} & \text{odd} \\ 0 & \text{even} \end{cases}$ (large N)

Th. $U \sim \mathcal{F}^1$ (even)

Even case : $\downarrow$ spinors reducible for even d.

sp. of self-adj ops

nonselfadj ops

$\mathcal{F}$: $\quad \pi_0(\mathcal{F}) \overset{\text{index}}{\cong} \mathbb{Z}$

$\pi_i(\mathcal{F}) \begin{cases} \mathbb{Z} & \text{even} \\ 0 & \text{odd} \end{cases}$

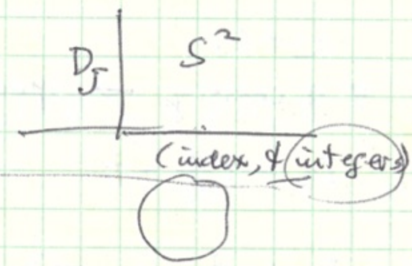
homot. gr.

$\Omega(U) \sim \mathcal{F} \qquad\qquad \mathcal{F}' \sim \Omega(\mathcal{F})$

↳ loop sp. $\qquad\qquad \mathcal{F} \sim \Omega(\mathcal{F}')$

Ex. 2-par. family $\qquad \pi_2(\mathcal{F}) = \mathbb{Z}$

D_J (even d.)



Finite # of pts in par. sp ← where op. is not invertible.

$\mathcal{F} \sim$ Line bundle $\to \mathbb{Z}$

det( ) = sec. of line bundle if anomaly

t' Hooft Quant Structure Black Hole

decay of bl. hole ?
↓
~~?~~ discrete spec. ?

1. Thermo $\rho(M) \sim e^{M^2}$: $S = 4\pi M^2$

$+ \circ \rightarrow$ $\sigma = |T|^2 \rho$

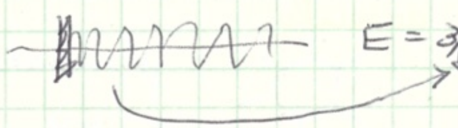
$\rightsquigarrow \circ$ $W = |T|^2 \rho_1 \rho_2$

$\rightarrow \dfrac{\rho_2(\,)}{\rho_1(\,)} = e^{\beta}$

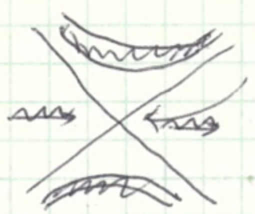
Problem. : steady state in & out particles
→ infinity of particles on horizon.

brickwall model

sol. of Klein-Gordon



$E = 3/\beta \; M_0$



worm hole → double our world in QM.
2 divergences runaway sol.

Can one restore QM coherence ?

Need some modifications ?

1. Equivalence th.
Totality of laws of physics → another H.

Rindler space :

$H = \int \mathcal{H} d^3x \qquad H' = \int \mathcal{H} \times z = \mathcal{H}_+ + \mathcal{H}_-$

$\mathcal{H}' \rightarrow A^+A - B^+B$

$H = \quad C^+C - D^+D \quad$: (Bogol. transf.)

1. $|0\rangle = \sum_n |n,n\rangle e^{-n\omega}$

2. $\sum |\underset{\cdot}{?}n\rangle e^{-n\omega} \quad \rightarrow \beta = 1/T = 2\pi$

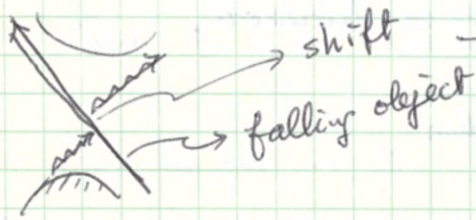
$|\psi\rangle \leftrightarrow \rho \qquad |\psi\rangle \leftrightarrow |n\rangle_{I} |m\rangle_{II} \sim |n\rangle_{I} \langle m_{II}| \sim \rho$

$|0\rangle \rightarrow C\, |n\rangle e^{-\pi n\omega} \langle n| \quad \rightarrow \beta = 1/T = \pi$

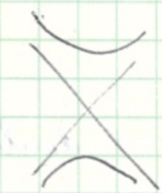
$\rightarrow T = 2T_{Hawk.}$

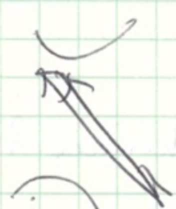
open question.

2. Better th ?



$\longrightarrow$ future horiz $\neq$ past hor.





J. Schwarz Superstr.

1. string (1-dim)
2. $D=10$.
3. Super
4. Finite & anomaly-free
5. No arbitrary dim. param.

Sust. $D=3,4,6,10$ in classical
$\hookrightarrow$ q.m.

$N=1$, Q^L Majorana-Weyl (16 comp) $\rightarrow G = SO(32)$ or $E_8 \times E_8$

$N=2A$ Q_1^L, Q_2^R l-r sym. $N=8$

$N=2B$ Q_1^L, Q_2^L $N=8$

$\rightarrow$ No YM.

6. Anomaly

$N=1$ YM bad anom

$N=1$, SG grav. anom.

$N=1$, YM+SG both anom.

$N=2A$ SG: trivial

$N=2B$ SG nontrivial.

Effective action analysis

$$e^{iS_{eff}} \sim \int D\varphi \, e^{i\mathcal{L}}$$

$S_{eff} = (N=1, D=10)$ SYM + ~~SG~~ SG + ... higher dim terms

SYM A, χ

SG $g_{\mu\nu}, B_{\mu\nu}, \phi, \psi_{\mu_L}, \lambda_R$

$A = A^i_\mu dx$ adj.

$\omega = \omega_\mu dx^\mu$ 10×10

$F = dA + A^2$ $R = d\omega + \omega^2$

gauge

$\delta A = \ldots$

Chern-Simons

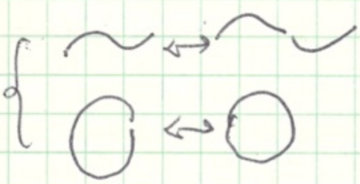
Anomaly cancel: $c\,\mathrm{Tr}F^6 \to c = 0$

Factorization cond → cancelled by extra term.

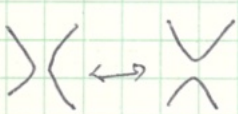
B must transf covariantly. $\sim B_2 X_8$

Type I SuSt

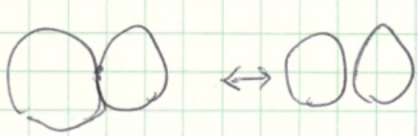
 x^μ, θ^α

 $g I^{YM}$

$g I$

 , etc. κI

Type II.



heterotic. YM, SG same const.

$$g_Y = \frac{g_{10}}{\sqrt{V_a}} \quad \kappa = \frac{\kappa}{\sqrt{V_a}}$$

$$\cancel{g_{10}^2} \quad \underline{\sqrt{\alpha'} = \text{Plank}} \rightarrow \cancel{g_Y^2}$$

$$\frac{\sqrt{V}}{\alpha'} \approx \text{Planck length}$$

M. Green

No general principles. $\exists$ now

Dynamics of a simple string $\quad X^\mu, \theta^\alpha$

$$\delta\theta^A = l_{mn}\gamma^{mn}\theta^A + \varepsilon^B$$

$$\delta X^\mu = l^\mu{}_\nu X^\nu + 2\bar{\varepsilon}\gamma^\mu\theta^A + a^\mu +$$

$$S = S_1 + S_2$$

$$S_1 = \frac{T}{2}\int \sqrt{g}\, g^{\alpha\beta}\, \Pi^*_{\alpha\mu}\Pi_{\beta\nu}\, d\xi^a \qquad \Pi : \text{susy}(X^\mu)$$

$$S_2 = -iT\int \epsilon^{\alpha\beta}(\theta\ \theta)\,\partial_\alpha X^\mu\,\theta\,(\theta\theta)^2$$

$\hookrightarrow \Rightarrow$ extra local inv. $\rightarrow$ free field th. in $D=10$.

$$\sim S_1 = \ \text{tr}\,(h^{-1}\partial_\alpha h)(h^{-1}\partial_\beta h)\sqrt{-g}\, g^{\alpha\beta}$$

$$S_2 = \int \epsilon^{\alpha\beta\gamma}\,\text{tr}[(\ )(\ )(\ )]$$

$\Big\} \rightarrow$ geometric

heterotic: similarly done

Type II not clear.

Quantization: ?? (Kaku?)

Vertex op. (V_B, V_F) susy $\rightarrow$ determined.

$$V_F \sim \bar{\theta}\gamma_\mu\left(g_F^{\mu\nu}P_\nu\theta - ih_\nu{}^{\mu\nu}\bar{\theta}\gamma_\mu\theta\right)e^{-ik\cdot x}$$

$$V_B = J_\mu\left(g_B P_\nu - i\, h_\nu{}^{\mu\nu}\bar{\theta} \cdots\right) \ ''$$

fns g_F, g_B, h_F, h_B determined by

$$\delta V_F = V_B \qquad \delta V_B = V_F$$

? Q. Neveu-Schwarz ~~not~~ = above form to higher orders

Light cone gauge
String Field Theory (Siegel?)

$\Phi[x(\sigma)\ \theta(\sigma)\ \bar\theta(\sigma), p^+]$

$$G_2 = \int DxD\theta D\tilde\theta\, dp^+\ \Phi_{p^+} g\, \Phi$$

$\hookrightarrow$ Laplace ϕ.

$\hookrightarrow$ $-\Box + \frac{N_{tot}}{\alpha'}$ + ferm.
Bose

→ nonlinear ~~gen.~~ real. of superPoincaré.

$$G = G_2 + gG_3 + \dots \qquad G_3 = \int \Pi\, DxD\tilde\theta\, dp^+$$

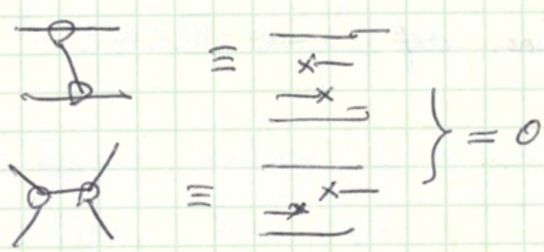
$$\times \Delta(3-2-1)\ \hat{G}(\sigma=\sigma_I)\ \Phi(1)\Phi(2)\Phi(3)$$
+perm.

↓ local op. at junction

No ~~#~~ higher other terms.

Closure of Poincaré alg.

$$\{Q_3^A, Q_3^B\} = 2\delta^{AB}H_4 = 0 :$$



breaks down for forward scatt.

Low energy limit.

Open str. $S \sim \int \dots \sum \frac{1}{2}\phi_{\{n\}}\left(-\Box + \frac{N}{\alpha'}\right)\Phi$

$$-g\int \dots \sum C_{lmn}\phi_l\phi_m\phi_n$$

↓ selec. rule $\delta(\sum p - \sum N_i)$

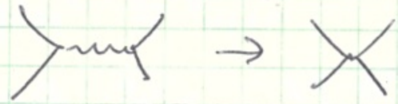
$\alpha' \to 0$ rescale $\phi \to \frac{\phi}{\sqrt{\alpha' M}}$ $(N \neq 0)$

$$\phi\Box\phi - i\sum\phi_n^2 + g\,\phi_0^3 C \dots + \sum g\frac{C}{\sqrt{M}}\phi_0^2\phi_n$$

→ quartic term

Closed str. case: two sets of terms

Cubic int:



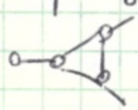
Black hole?

Q. (Banks) What is the residual gauge sym?

V. Kac 202

Construction of basic reps of $E_8^{(1)}$

Kac-Moody alg.



$a_{ii} = 2$ A: Cartan matr.

$a_{ij} = \begin{cases} 0 \\ -1 \end{cases}$

Root lattice Q basis h_i

$W(\Gamma)$ Weyl gr. fundam rep of G: $h_i \to a_{ij} h_j$

3 types

1. Finite $\Leftrightarrow$ Q pos. def. $\rightarrow$ $W < \infty$

$A_n,\ D_n,\ E_6\ E_7\ E_8$

n = 3 2

dim W. n+1 4 3 2 1

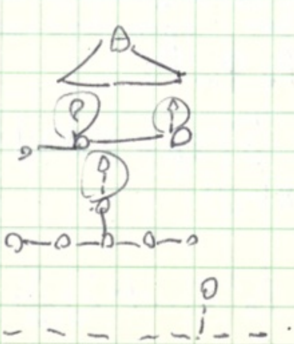
2. Affine type $\Leftrightarrow$ pos. semidefi $\longleftrightarrow$ $\exists$ labeling s.t. label = Σ sum neighbor

A_n

D

E

E_8



3 Wild type E_{10} 

K-M alg. Lie al on Chevalley gen. e_i f_i h_i,

$[e\ f] \to h$

$[h, e] \to e$, $[h, f] \to f$

$[e, e] \to 0$ $[f, f]$ if $a_{ij} = 0$

$[[e\ e]e] = 0 = [[f f] f]$ if $a_{ij} = 0$

$\Rightarrow$ 1. $\dim g$ ~~$\Leftrightarrow$~~ $< \infty \Leftrightarrow \Gamma$ finite

2. expon. growth ∞

:

Highest wt rep: $\lambda = \{\lambda_i\}$

$\{L(\lambda), \pi\}$: $\underbrace{\cancel{\ }}_{\pi(e_i) v_\lambda} = 0$, $\overset{\pi(h_i)}{\cancel{\ }} v_\lambda = \lambda_i v_\lambda$

unitary: all $\lambda_i \in \mathbb{Z}_+$

Th. $\mathrm{chr}(\lambda) = \sum \pm e^{\cdot\cdot} / \sum \cdots$

1. Finite type: unitary $L(\lambda)$ = all irrep.
2. Affine type: basic reps.

$$\begin{cases} \lambda_0 = 1 & (0\text{-th} \\ \lambda_i = 0 & \text{for } i \neq 0 \end{cases}$$

$E_8^{(1)}$: 1 

all possible ways to constr. basic reps.

Explicit constr E_8 (Frenkel-Kac) 1980 Ann Math

$\sum$ coords $\in 2\mathbb{Z}$,

$\mathbb{Z}$ or $\frac{1}{2}\mathbb{Z}$

roots $\Delta = \alpha + Q\{(\alpha,\alpha)=2\}$

$\Delta = \{\pm\varepsilon_i \pm \varepsilon_j + \cdots \}$

$[\alpha,\beta]=0$ $\alpha, \beta \in Q$

$e_\alpha, e_\beta = 0$ if $\alpha+\beta \in \Delta \cup \{0\}$

$E_8^{(1)} = \mathbb{C}[t,t^{-1}] \otimes E_8 + \mathbb{C}c$ $\quad E, F, H$: Chevalley generators

$e_i = 1 \otimes E_i$ etc

Basic rep. V_{II} 1) unitary

2) center $c = 1$ (in gen. $c = 1, 2, 3, \ldots$)

$E_8^{(1)} \supset E_8$ commutes with d.

Eigenspace decomp

$V = \sum V_k$, is E_8 inv.

$\operatorname{ch} V \overset{\text{def}}{=} \sum (t_V e^\alpha) q^k = \sum e^{(\gamma|h)} q^{\frac{1}{2}|\gamma|^2} / \prod (1-q) \cdots$

$h=0 \Rightarrow$ part. fn.

$q^{-1/2} \sum \dim V_k \, q^k = \sqrt[3]{j(\,)}$

$\downarrow$

mod. inv.

V is irred.

$s = \sum (t^k \otimes h) + \mathbb{C}i$ Heisenberg subalg.

$\hat{S} \to S = H(\mathbb{C}[t,t^{-1}])$ Infinite Heisenberg

$V = \mathbb{C}[Q] \otimes S(s)$

E_8^{i} acts on V : creation γ_k

S: vertex op.

$\# W(E_8) = 2^{14} \cdot 3^5 \cdot 5^2 \cdot 7$ Aut Q on

112 conj classes

30 non-deg. class $\rightarrow$ 9 primitive classes

$L \stackrel{def}{=} \{ h \in \underset{\sim}{h} \mid w \cdot h = h + Q \}$ $\det(1-w) \neq 0$ $\det(1-w) = 1$

Heisenberg finite, inf; & Lie group type