Consideration of vacuum energy

$$H = H_0 + \underset{A}{\frac{G^2}{2}\phi^4} - \underset{B}{G^2\lambda v^2\phi^2} + \underset{C}{f\phi\cdot\bar{\psi}\psi} \qquad f = \alpha G$$

$$G\frac{\partial}{\partial G}\langle H\rangle = G\left\langle \frac{\partial H}{\partial G}\right\rangle = 2\langle A\rangle + 2\langle B\rangle + \langle C\rangle$$

This does not help much. Better to compute it directly.

Effect of renormalization

$$V = \frac{G^2}{2}(\sigma^2+\pi^2)^2 + \frac{G^2}{2}(2v)^2\sigma^2 + 2vG^2\sigma(\sigma^2+\pi^2)$$

Consider σ^4 term $\rightarrow \binom{4}{2}\langle\sigma^2\rangle\sigma^2 = 6\langle\sigma^2\rangle\sigma^2$

$$\sim \delta m_\sigma^2\sigma^2$$

$$\langle\sigma^4\rangle = 3\langle\sigma^2\rangle\langle\sigma^2\rangle$$

so $-\langle\delta m_\sigma^2\sigma^2\rangle = -2\langle\sigma^4\rangle$

Similarly $\sigma^2\pi^2 \rightarrow \langle\sigma^2\rangle\pi^2 + \langle\pi^2\rangle\sigma^2$

$$\sim \delta m_\pi^2\pi^2 + \delta m_\sigma^2\sigma^2$$

$$\langle\sigma^2\rangle\langle\pi^2\rangle = \frac{1}{2} \text{ of } \delta m_\pi^2\langle\pi^2\rangle + \delta m_\sigma^2\langle\sigma^2\rangle$$

$f\bar{\psi}\psi \cdots\cdots \bar{\psi}\psi f$ gives the same relation.

$\Rightarrow$ After separating out the self-energies,

the vacuum bubbles change sign.

For the purpose of computing the interaction energy, one may also take ½ of the self-energies.

80. Check of the prescription for the case of 4-fermion interaction. $\sim -\frac{g}{2}(\bar\psi\psi\bar\psi\psi + \gamma_5 \cdot \gamma_5)$

? The vac. energy* $= -\frac{1}{\cancel{4}}g\langle\bar\psi\psi\rangle\langle\bar\psi\psi\rangle$

$$= -\frac{1}{\cancel{4}g}(g\langle\bar\psi\psi\rangle)^2$$

$$\hookrightarrow = m_f^2 = \left[g\left(\Lambda^2 - m_f^2 \ln\frac{\Lambda^2}{m_f^2}\right) \times 1/4\pi^2\right]^2 m_f^2$$

$$1 = \frac{g}{4\pi^2}\left(\Lambda^2 - m_f^2 \ln\frac{\Lambda^2}{m_f^2}\right)$$

$$\text{or} \quad \frac{1}{g} = \frac{1}{4\pi^2}\left(\Lambda^2 - m_f^2 \ln\frac{\Lambda^2}{m_f^2}\right)$$

$$\text{vac. energy} = -\frac{m_f^2}{16\pi^2}\left(\Lambda^2 - m_f^2 \ln\frac{\Lambda^2}{m_f^2}\right)$$

Fermion zeropt energy : $-E = -(k^2+m_f^2)^{1/2}$

* This is not right: $g^2\frac{\partial\langle H\rangle}{\partial g} = -g^2\langle\bar\psi\psi\rangle\langle\bar\psi\psi\rangle/2$

$$= -m_f^2/\cancel{4}2$$

$$d\left(\frac{1}{g}\right) = -\frac{1}{4\pi^2}\, dm_f^2\left(\ln\frac{\Lambda^2}{m_f^2} - 1\right)/2$$

$$\rightarrow \quad d\langle H\rangle = -m_f^2\frac{1}{4\pi^2}\, dm_f^2\left(\ln\frac{\Lambda^2}{m_f^2} - 1\right)/2$$

$$\langle H\rangle = -\frac{1}{4\pi^2}\left[\frac{1}{2}m_f^4\left(\ln\frac{\Lambda^2}{m_f^2} - 1\right) + \frac{1}{4}m_f^4\right]/2$$

$$= -\frac{1}{\cancel{8}16\,\pi^2}m_f^4\left(\ln\frac{\Lambda^2}{m_f^2} - \frac{1}{2}\right)$$

Compute also $\langle \alpha\cdot p\rangle + \frac{1}{2}\langle m\beta\rangle$

$$= -\frac{p^2}{E} - \frac{1}{2}\frac{m^2}{E}$$

$$\frac{1}{E} = \frac{1}{p} - \frac{1}{2}\frac{m^2}{p^3} + \frac{3}{8}\frac{m^4}{p^5} - \cdots$$

$$\to -\int\left[p - \frac{1}{2}\frac{m^2}{p} + \frac{3}{8}\frac{m^4}{p^3} + \frac{m^2}{2p} - \frac{m^4}{4p^3}\cdots\right] d^3p \times \frac{2}{(2\pi)^3}$$

$$= -\frac{1}{4\pi^2}\Lambda^4 - \frac{1}{8}m^4 \cdot \frac{1}{\pi^2}\ln\frac{\Lambda}{m_f}$$

$$= -\frac{1}{4\pi^2}\Lambda^4 - \frac{m^4}{16\pi^2}\ln\frac{\Lambda^2}{m_f^2} \qquad \text{O.K.}$$

Case of 3-vertices

$$\sigma^3 \to \frac{1}{2}\sigma\langle\sigma^2 \quad \sigma^2\rangle\sigma \to 2\cdot 9\cdot\frac{1}{2}\sigma\langle\sigma\ \sigma\rangle^2\sigma$$

$$\to \text{vac.} = \frac{18}{2}(\langle\sigma\ \sigma\rangle)^3$$

H_{int} From $\frac{1}{2}\langle\sigma^3 \quad \sigma^3\rangle \to \frac{1}{2}3\cdot 2\,(\langle\sigma\ \sigma\rangle)^3$

$$\sigma\pi^2 \to \frac{1}{2}\sigma\langle\pi^2\ \pi^2\rangle\sigma \to 2\cdot\frac{1}{2}\sigma\langle\pi\ \pi\rangle^2\sigma$$

$$\to \frac{2}{2}\cdot\langle\sigma\sigma\rangle\langle\pi\pi\rangle^2$$

$\frac{1}{2}$ also $\to \frac{1}{2}\pi\cdot\langle\sigma\pi \quad \sigma\pi\rangle\pi \times 4 \to \frac{4}{2}\pi\langle\sigma\sigma\rangle\langle\pi\pi\rangle\pi$

$$\to \frac{4}{2}\langle\sigma\sigma\rangle\langle\pi\pi\rangle^2$$

$H_{int} = \frac{1}{2}\langle\sigma\pi^2 \quad \sigma\pi^2\rangle \to \frac{1}{2}\cdot 2\cdot\langle\sigma\sigma\rangle\langle\pi\pi\rangle^2$

So in this case $H_{int} = \frac{1}{3}\langle H_{self}\rangle$

82.

Problem with Minkowski vs ~~Lorentz~~ Euclidean metric.

M: $i\int \frac{d^4k}{k^2-m^2+i\varepsilon} = \pi^2(\Lambda^2 - m^2 \ln \ )$ $\qquad d^4k = \pi^2 k^2 dk^2$

but $i\int \frac{k^2}{k^2-m^2+i\varepsilon} d^4k \to i\int d^4k - i\int \frac{m^2}{k^2-m^2+i\varepsilon} d^4k$

E: $\int \frac{d^4k}{k^2+m^2} d^4k \to \pi^2(\Lambda^2 - m^2 \ln)$

$\int \frac{k^2}{k^2+m^2} d^4k \to \int d^4k - \int \frac{m^2}{k^2+m^2} d^4k$

So one should use Euclidean form, or translate
$i\int d^4k \to \int d^4k$

Another problem:

$\int \ln \frac{\Lambda^2}{ck^2-m^2+i\varepsilon} d^4k$ should pick up a branch cut.

$\frac{\partial}{\partial m^2} = \int \frac{1}{ck^2-m^2+i\varepsilon} d^4k \to -i\pi \int \delta(ck^2-m^2)\, d^4k$

$\to \int \ln \quad = -i\pi \int_{\infty}^{m^2} \delta(ck^2-m^2)\, d^4k\, dm^2$

$= i\pi \int d^4k\, \theta(-ck^2+m^2)$

$= i\pi \cdot \pi^2 \int k^2 dk^2\, \theta(\quad)$?

too many π's.

$\int \frac{1}{k^2-m^2+i\varepsilon} d^4k = -i\pi^2 (\Lambda^2 - m^2 \ln \frac{\Lambda^2}{m^2})$

$\int^{m^2} dm^2 \quad '' \quad -i\pi^2 (\Lambda^2 m^2 - \frac{m^4}{2} \ln \frac{\Lambda^2}{m^2} - \frac{m^4}{4}) \Big|^{m^2}_{?}$

Eucl. result

$$\int \ln \frac{\Lambda^2}{k^2+m^2}\, d^4k = \pi^2 \int k^2 \ln \frac{\Lambda^2}{k^2+m^2}\, dk^2$$

$$= \pi^2 \Big[\frac{1}{2} k^4 \ln \frac{\Lambda^2}{k^2+m^2}\Big| + \frac{1}{2}\int \frac{k^4}{k^2+m^2}\, dk^2 \Big]$$

$$= \pi^2 \Big[\underset{0}{\downarrow} \quad \frac{1}{2}\int k^2 dk^2 - \frac{1}{2}\int \frac{m^2 k^2}{k^2+m^2}\, dk^2 \Big]$$

$$= \pi^2 \Big[\frac{1}{4} k^4 \Big| - \frac{1}{2} m^2 k^2 \Big| + \frac{1}{2} m^4 \int \frac{dk^2}{k^2+m^2} \Big]$$

$$= \pi^2 \Big[\underbrace{\frac{1}{4}\Lambda^4 - \frac{1}{2} m^2 \Lambda^2}_{\text{ambiguous.}} + \frac{1}{2} m^4 \ln \frac{\Lambda^2}{m^2} \Big]$$

I forgot $\frac{1}{2} k^4 \ln \frac{\Lambda^2}{k^2+m^2}\Big|_0^{\Lambda^2} \to \frac{1}{2}\Lambda^4 \ln \frac{1}{1+\frac{m^2}{\Lambda^2}}$

$$\to -\frac{1}{2}\Lambda^2 m^2$$

so $= \pi^2 \Big[\frac{1}{4}\Lambda^4 - m^2\Lambda^2 + \frac{1}{2} m^4 \ln \frac{\Lambda^2}{m^2} \Big]$

$\downarrow$

This agrees with $\frac{\partial}{\partial m^2}$ calculation.

When $\ln \frac{\Lambda^2}{k^2+m^2} \to \ln \frac{\Lambda^2}{\alpha\beta k^2+m^2}$, then one must first integrate over α, since $\alpha\beta$ can go to 0.

presence or absence of $m^2\Lambda^2\ln\Lambda^2$ terms in vac. energy.

Conclusion:  there do not have them.

To show this in various ways:

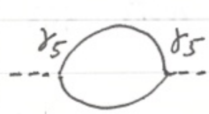
Use of ~~dis~~ dispersion relations.

* 

$$\int \frac{\frac{K^2}{2}-2m^2}{k^2-K^2}\sqrt{1-\frac{4m^2}{K^2}}\;dK^2$$

$k\to 0$:
$$-\sim\int\left(\frac{dK^2}{2}-2m^2\frac{dK^2}{K^2}-m^2\frac{dK^2}{K^2}\right)$$
$$\sim -\frac{\Lambda^2}{2}+3m^2\ln\frac{\Lambda^2}{4m^2}$$

k-dependence
$$-\int\frac{1}{2}k^2\frac{dK^2}{K^2}\to -\frac{1}{2}k^2\ln\frac{\Lambda^2}{4m^2}$$



$$\int\frac{\left(\frac{K^2}{2}\right)}{k^2-K^2}\sqrt{}\;dK^2$$
$$\sim -\frac{\Lambda^2}{2}-\frac{1}{2}k^2\ln\frac{\Lambda^2}{4m^2}+m^2\ln\frac{\Lambda^2}{4m^2}$$

These integrals are ~~sm~~ different from Feyman integrals:
$$\Lambda^2\to\frac{1}{2}\Lambda^2$$

* $p\cdot p'\to -\frac{K^2}{2}+m^2$

For unequal masses

$$\sqrt{} \to \frac{|\vec{p}|}{k} \qquad |p| = \sqrt{\frac{(k-m_1-m_2)(k+m_1+m_2)(k-m_1+m_2)(k+m_1-m_2)}{k^2}}$$

For $m_2 = 0$ $\qquad \sqrt{} \to \left(1 - \frac{m_1^2}{k^2}\right)$

$$\int \frac{\frac{k^2}{2}}{k^2 - k^2} \sqrt{1 - \frac{4m_f^2}{k^2}} \; \frac{1}{k^2 - m_\pi^2} \; d^4k$$

$\hookrightarrow = 0$

$$\to \iint \frac{k^2}{2} \sqrt{} \cdot \frac{1}{-k'^2} \left(1 - \frac{k^2}{k'^2}\right) dk^2 dk'^2$$

$k^2: \quad 4m_f^2 \to k'^2$

$k'^2: \quad 4m_f^2 \to \Lambda^2$

$$\iint \sim -\frac{1}{2} \iint \frac{k^2}{k'^2} \left(1 - \frac{2m_f^2}{k^2}\right)\left(1 - \frac{k^2}{k'^2}\right) dk^2 dk'^2$$

$$\to -\frac{1}{2} \int \left(\frac{1}{2} k^4 - 2m_f^2 k^2 - \frac{1}{k'^2} \cdot \frac{1}{3} k^6 + m_f^2 \frac{k^4}{k'^2} \right) \Big|_{4m_f^2}^{k'^2} \frac{dk'^2}{k'^2}$$

$$\cancel{= -\frac{1}{2} \int \Big[\frac{1}{2} k'^4 - \frac{1}{2}(4m_f)^2 - 2m_f^2 (k'^2 - 4m_f^2) - \frac{1}{3}\left(k'^2 - \frac{(4m_f^2)^3}{k'^2}\right)}$$

$$\cancel{+ m_f^2 \left(k'^2 - \frac{(4m_f^2)^2}{k'^2}\right) \frac{dk'^2}{k'^2}}$$

$$\cancel{= -\frac{1}{2} \Big[\frac{1}{4}(\Lambda^4 - (4m_f^2)^2) - \frac{1}{2}(4m_f^2)^2 \ln \frac{\Lambda^2}{4m_f^2}}$$

$$\cancel{- 2m_f^2 \int \frac{dk'^2}{k'^2} \ln \frac{k'^2}{4m_f^2}}$$

$\exists$ no $m_f^2 \ln \Lambda^2/m_f^2$ terms.

The vacuum energy consists of

1.  free zeropt en.

2. $\langle \lambda v^2 (\sigma^2 + \pi^2)/2 \rangle$

3. $-\delta m_\sigma^2 \langle \sigma^2/2 \rangle - \delta m_\pi^2 \langle \pi^2/2 \rangle - \delta m_f \langle \bar\psi \psi \rangle$

4.   etc.

Now. $\delta m_\pi^2 = 0$

δm_σ^2 contains ln only.

$$-\delta m_f \langle \bar\psi \psi \rangle \to + \delta m_f \left(\Lambda^2 - m_f^2 \ln \frac{\Lambda^2}{m_f^2} \right) m_f$$

and $\delta m_f \sim f^2 m_f \ln \frac{\Lambda^2}{m_f^2}$ for π^0

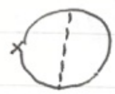
~~con~~ Similarly for γ & gluon contributions.

Q. What is the rôle of 2. & 3. ?
Are they supposed to cancel 1:  δm^2 ?

In that case there should be no $\Lambda^2 \ln \Lambda^2$ terms anyway.

Radiative corrections.

compute 2σ correction to the tadpole

 $= \frac{\partial}{\partial m_f}$  $\times \frac{1}{2}$

From p.84

$$= \int \frac{d^4k}{k^2-m_\sigma^2}\frac{\partial}{\partial m}\int_{4m^2}^{\Lambda^2} \frac{\frac{\kappa^2}{2}-2m^2}{k^2-\kappa^2}\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2$$

$$= \quad '' \quad \left(\frac{-4m}{k^2-m^2}\sqrt{} + \left(\frac{\kappa^2}{2}-2m^2\right)\frac{-\frac{4m}{\kappa^2}}{\sqrt{}} \right) d\kappa^2$$

$$= \int \quad '' \quad \int \frac{1}{k^2-\kappa^2}\frac{1}{\sqrt{}}\left[-4m\left(1-\frac{4m^2}{\kappa^2}\right) - 4m\left(\frac{1}{2}-\frac{2m^2}{\kappa^2}\right)\right]$$

$$= \quad '' \quad '' \quad \int \frac{1}{\sqrt{}}(-4m)\frac{3}{2}\left(1-\frac{4m^2}{\kappa^2}\right) d\kappa^2$$

$$= -6m \int \frac{d^4k}{(k^2-m_\sigma^2)(k^2-\kappa^2)}\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2$$

$$= -6m. \int_{\kappa^2}^{\Lambda^2} \frac{d\kappa'^2}{l^2-\kappa'^2}\int_{4m^2}^{\Lambda^2} \frac{\bar{p}}{\kappa'}\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2 \Big|_{l=0.}$$

$$\frac{\bar{p}}{\kappa'} = 1-\frac{\kappa^2}{\kappa'^2} \quad \text{if } m_\sigma = 0. \qquad (p.85)$$

$$\text{so} \quad = \frac{1}{2}\times 6m. \int_{\kappa^2}^{\Lambda^2}\left(1-\frac{\kappa^2}{\kappa'^2}\right)\frac{d\kappa'^2}{\kappa'^2}\int_{4m^2}^{\Lambda^2}\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2$$

1st ∫al   2nd ∫al

$$\int\left(1-\frac{\kappa^2}{\kappa'^2}\right)\frac{d\kappa'^2}{\kappa'^2} = \ln\frac{\Lambda^2}{\kappa^2} + \frac{\kappa^2}{\kappa'^2}\Big|_{\kappa^2}^{\Lambda^2} = \ln\frac{\Lambda^2}{\kappa^2} + \frac{\kappa^2}{\Lambda^2} - 1$$

$$\int\left(\ln\frac{\Lambda^2}{\kappa^2} + \frac{\kappa^2}{\Lambda^2} - 1\right)\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2$$

$$= \int\left(\frac{1}{\kappa^2}\int\sqrt{}\right)d\kappa^2 + \int\left(\frac{\kappa^2-\Lambda^2}{\Lambda^2}\right)\sqrt{1-\frac{4m^2}{\kappa^2}}\, d\kappa^2$$

$$\int \sqrt{1-\frac{4m^2}{k^2}}\,dk^2 = 2\int \sqrt{k^2-4m^2}\,dk = \cancel{8}4m^2\left(\frac{1}{2}\,\mathrm{sh}\,2\theta - \theta\right)$$

$\mathrm{ch}\,\theta = k/2m$

$$= 4m^2\left[\frac{k}{2m}\sqrt{\left(\frac{k}{2m}\right)^2-1} - \ln\left(\frac{k}{2m}+\sqrt{}\right)\right]$$

$$= k^2\sqrt{1-\frac{4m^2}{k^2}} - 4m^2\ln\left(\frac{k}{2m}+\sqrt{}\right)$$

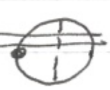
So $\int$ of p.87

1st term $=:$ $\displaystyle\int_{4m^2}^{\Lambda^2}\sqrt{}\,dk^2 - 4m^2\int\frac{dk^2}{k^2}\ln\left(\frac{k}{2m}+\sqrt{}\right)$

$$\cong \Lambda^2\sqrt{\left(1-\frac{4m^2}{\Lambda^2}\right)} - 4m^2\cdot\left(\ln\frac{\Lambda}{m}\right)^2 \quad \hookrightarrow \quad m^2\left(\ln\left(\frac{\Lambda^2}{m^2}\right)\right)^2$$

$$- 4m^2\ln\left(\frac{\Lambda}{m}\right)$$

$$\hookrightarrow \quad 2m^2\ln\left(\frac{\Lambda}{m}\right)^2$$

~~Finally~~ 

$$\cancel{= -6m\left(\Lambda^2 - 2m^2\ln\frac{\Lambda^2}{m^2}\right.}.$$

2nd term: $\displaystyle\int\left(\frac{k^2}{\Lambda^2}-1\right)\sqrt{}\,dk^2$

$$\sim \frac{1}{\Lambda^2}\frac{k^4}{2}\Big| - \int\sqrt{}\,dk^2$$

This cancels part of the above. $\sim$

$$\hookrightarrow \quad \frac{1}{2}\Lambda^2$$

So  $= \frac{1}{2}6m^2\left(\frac{1}{2}\Lambda^2 - m^2\left(\ln\frac{\Lambda^2}{m^2}\right)^2\right)$

translate to Feynman integral Λ (p.87) $\rightarrow$ $\frac{1}{2}6m\left(\Lambda^2 - m^2\left(\ln\frac{3\Lambda^2}{m^2}\right)^2\right)$

The linear $\ln\frac{\Lambda}{m}$ term is ambiguous, may be absorbed into $\left(\ln\left(\frac{\Lambda c}{m}\right)\right)^2$

One must also compute

Correction: if one renormalizes the loop the $\int$ of p.87 changed to

$$= (k^2-m_\sigma^2)^2 \int \frac{\frac{K^2}{2}-2m^2}{k^2-K^2} \frac{1}{(m_\sigma^2-K^2)^2} \sqrt{}\, dK^2$$

$\frac{1}{2}\frac{\partial}{\partial m_\sigma}$ changes the kernel by $\left(\frac{k^2-m_\sigma^2}{m_\sigma^2-K^2}\right)^2$

multiply by $\frac{1}{k^2-m_\sigma^2}$ $\cancel{\to \iint \frac{k^2-m_\sigma^2}{m_\sigma^2-K^2} \frac{\frac{K^2}{2}-2m^2}{k^2-K^2} \sqrt{}\, \frac{p}{K'}\, dK^2 dK'^2}$

$$\to \int \frac{k^2-m_\sigma^2}{k^2-K^2}\, d^4k \times \frac{\cancel{\frac{K^2}{2}-2m^2}\ 1}{(m_\sigma^2-K^2)^2} \sqrt{}\, dK^2 \times (-3m)$$

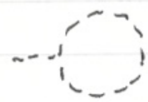
This $\int$al is ambiguous. But one expects to get $\Lambda^2 \ln \Lambda$ terms.

Interpretation of $\Lambda^2 \ln\Lambda$ terms

 use dressed propagator, then

$$\frac{1}{L} \to \frac{1}{Z_2}\,\frac{1}{\gamma\cdot p - M}$$

It is easier for bosons

 $\to$ Lehmann rep. $\left\langle \int \frac{\rho}{k^2-\kappa^2}\, d\kappa^2 \right\rangle$

$$\to \Lambda^2 \int \rho \, d\kappa^2 = Z_3^{-1}\Lambda^2$$

For fermions

$$\frac{1}{L} = \int \frac{\rho_1 \gamma\cdot p + \rho_2 m}{p^2-\kappa^2}\, d\kappa^2$$

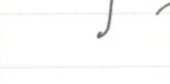
$$\int \rho_1 = Z_2^{-1} \qquad \int \rho_2 = ? \quad Z_2^{-1} m_0 \;?$$

$$m_0 = f_0 v$$

I would like to know the sign of $\Lambda^2 \ln\Lambda$

For boson, the self-en. has the repr.

$$(k^2-m_\sigma^2)^2 \int \frac{\rho}{k^2-\kappa^2}\,\frac{d\kappa^2}{(m_\sigma^2-\kappa^2)^2}$$

$\to$  loop $= \displaystyle\int \frac{\rho}{(\kappa^2-m_\sigma^2)^2}\, d\kappa^2$

$$\rho = \left(\tfrac{1}{2}\kappa^2 - 2m_\sigma^2\right)\sqrt{1-\frac{4m^2}{\kappa^2}}$$

$$\int \sim \frac{1}{2}\int \frac{d\kappa^2}{\kappa^2} = \frac{1}{2}\ln\frac{\Lambda^2}{4m^2} > 0$$

$\therefore$ self-energy < 0 : ~~opposite to~~

$$\Lambda^2 - m_\sigma^2 \ln - \delta m_\sigma^2 \ln$$

Renormalization of Λ^2

$$\Sigma c_i g_i^2 \rightarrow c_i g_i^2 Z_i^{-1} \qquad \text{except for } f^2 : \ \frac{m_0}{m} Z_f^{-1}$$

σ, π contributions to fermion self-energy $\propto f^2$

$$\begin{array}{lll} \delta Z & -\frac{1}{2}\ln \times 4 = -2\ln \\ \delta m & (3-1)\ln \quad = 2\ln \end{array} \Big> \text{ they cancel} \quad \begin{array}{l}\text{true only} \\ \text{if } f_1 = f_2\end{array}$$

fermion contributions to σ an π self-energy $\propto f^2$

Each  $\rightarrow \frac{12}{16\pi^2} \cdot \frac{1}{2} \ln \frac{\Lambda^2}{m^2}$

$m : \sim m_\sigma^2$ for σ
$\sim 4m_f^2$ for π^0
$\sim 4m_f^2$ $\quad \pi^\pm$

fermion contributions to gauge self-energy $\propto g^2$

So the f-sensitive terms are only from Higgs.

$$c_i g_H^2 \cdot \frac{3}{8\pi^2} \ln \frac{\Lambda^2}{m^2}$$

modified eq. I :

$$\sum c_i g_i^2 = -\frac{1}{16\pi^2} \sum_i^6 g_{H_i}^2 f^2 \ln \frac{\Lambda^2}{m_i^2}$$

Eq. II. $\quad \frac{\lambda}{2} g_H^2 = \frac{1}{16\pi^2} \sum_i d_i g_i^4 \ln \frac{\Lambda^2}{m_i^2}$

g_{H_i} runs over σ & π.

$g_\sigma^2 = \frac{3}{2} g_H^2$

$g_\pi^2 = \frac{1}{2} g_H^2$

$c_f = -12 \qquad d_f = -12$

$c_H = 3 \qquad d_H = 3/2$

$c_V = 3 \qquad d_V = 3$

Infrared sensitive terms in II: π^0

$$-\frac{3g_H^2}{16\pi^2} f^2 \ln\frac{\Lambda^2}{f^2 v^2}$$

so let $f \to f_1, f_2$ and $f_2 \ll f_1$.

From I: $-12(df_1^2 + df_2^2) = -\frac{3}{16\pi^2} g_H^2 d\left(f_2^2 \ln\frac{\Lambda^2}{f_2^2 v^2}\right)$

assumed $12 \cancel{\gg} > \frac{3g_H^2}{16\pi^2}\ln\frac{\Lambda^2}{f_1^2 v^2}$ $\quad *$

$$\left(\frac{-1}{4}\frac{g_H^2}{16\pi^2}\ln\frac{\Lambda^2}{f_1^2 v^2}+1\right) df_1^2 = \frac{1}{4}\cdot\frac{1}{16\pi^2} g_H^2\, d\left(f_2^2\ln\frac{\Lambda^2}{f_2^2 v^2}\right) - df_2^2$$

↳ ~ 1

From II: $d(\text{other}) + d_f\, d\left(f_1^4 \ln\frac{\Lambda^2}{f_1^2 v^2} + f_2^4 \ln\frac{\Lambda^2}{f_2^2 v^2}\right) = 0$

or $d_f\Big[2 f_1^2 \ln\frac{\Lambda^2}{f_1^2 v^2}\, df_1^2 + f_2^2 \ln\frac{\Lambda^2}{f_2^2 v^2}\, df_2^2$

$\qquad + f_2^2\, d\left(f_2^2 \ln\frac{\Lambda^2}{f_2^2 v^2}\right)\Big]$

$$+\underbrace{\sum d_i g_i^4 \frac{dv^2}{v^2}}_{0} \; \underbrace{\cancel{\cdot\left(f_1^4 \ln\frac{\Lambda^2}{f_1^2 v^2} + f_2^4 \ln\frac{\Lambda^2}{f_2^2 v^2}\right)\frac{dv^2}{v^2}}}_{\text{ignore}} = 0$$

For $df_1^2 \propto + d\left(f_2^2 \ln\frac{\Lambda^2}{f_2^2 v^2}\right)$ one must have

$$\frac{1}{4}\frac{1}{16\pi^2} g_H^2 \ln\frac{\Lambda^2}{f_2^2 v^2} > 1$$

compare with $*$:

$1 > \frac{1}{4}\frac{1}{16\pi^2}\ln\left(\frac{\Lambda^2}{f_1^2 v^2}\right)$, $\quad \frac{1}{4\cdot 16\pi^2}\ln\frac{\Lambda^2}{f_2^2 v^2} > \frac{1}{g_H^2}$

~~so $g_H^2 \gg 1$ which is O.K.~~ ↙

$$\frac{1}{4\cdot 16\pi^2}\left(\ln\frac{\Lambda^2}{f_1^2 v^2} + \ln\frac{f_1^2}{f_2^2}\right) > \frac{1}{g_H^2}$$

~~e.g. $g_H = 2$~~

or $\frac{1}{4\cdot 16\pi^2}\ln\frac{f_1^2}{f_2^2} > \frac{1}{g_H^2} - 1$

$f_1^2/f_2^2 > 1$ if $g_H^2 < 1$.

Now if all lighter fermions are included,

bottom + ~~t~~ $\tau + \nu$ = ~~3+3~~ $3+2 = 5$

1st & 2nd gen. $8 \times 2 = 16$ $\Big\} > 21$

$$\rightarrow \frac{1}{4 \cdot 16\pi^2} \times \frac{21}{3} = \frac{7}{64\pi^2} \sim \frac{1}{9\pi^2} \sim \frac{1}{100}$$

$$\ln \frac{f_1^2}{f_2^2} \gtrsim \frac{100}{g_H^2}\left(\frac{1}{g_H^2} - 1\right)$$

$$(\Sigma d_i g_i^4)\, \frac{dv^2}{v^2} \cancel{= d_f \cdot 2 f_1^2 \ln \frac{\Lambda^2}{f_1^2 v^2} \cdot \left[\left(\frac{g_H^2}{4 \cdot 16\pi^2} + f_2^2\right) d\left(f_2^2 \ln \frac{\Lambda^2}{f_2^2 v^2}\right) + f_2^2 \ln \frac{\Lambda^2}{f_2^2 v^2}\, df_2^2\right.}$$

$$= d_f \left[2 f_1^2 \ln \frac{\Lambda^2}{f_1^2 v^2} \left\{ \frac{g_H^2}{4 \cdot 16\pi^2}\, d\left(f_2^2 \ln \frac{\Lambda^2}{f_2^2 v^2}\right) - df_2^2 \right\} + f_2^2 \ln \frac{\Lambda^2}{f_2^2 v^2}\, df_2^2 + f_2^2\, d\left(f_2^2 \ln \frac{\Lambda^2}{f_2^2 v^2}\right) \right]$$

For small enough f_2^2, the dominant term $= -df_2^2$

$d_f = -12$, $\rightarrow$ sign of r.h.s. > 0 $\Rightarrow$ $dv^2 > 0$

as $df_2^2 > 0$.