June 90

Gauge field self-energies

$$G_{\mu\nu} = \partial_{[\mu} A_{\nu]} - \frac{g}{2} i\, [A_\mu, A_\nu] \qquad A = \vec{A} \cdot \vec{\tau}$$

SU(2) $\quad \vec{G}_{\mu\nu} = \partial_{[\mu} \vec{A}_{\nu]} + g \vec{A}_\mu \times \vec{A}_\nu$

$$L_G : \; -\frac{1}{4} (\partial_{[\mu} A_{\nu]})^2 - \frac{g}{2} \partial_{[\mu} \vec{A}_{\nu]} \cdot \vec{A}_\mu \times \vec{A}_\nu - \frac{g^2}{4} (\vec{A}_\mu \times \vec{A}_\nu)^2$$

$$L_{G\phi} : \; |D_\mu \phi|^2 = |\partial_\mu \phi|^2 + \frac{ig}{2} (\partial_\mu \phi^\dagger \tau \cdot A \phi - \phi^\dagger \tau \cdot A \partial_\mu \phi)$$
$$+ g^2 |\phi|^2 \vec{A}_\mu^2$$

Include U(1) :

$$-\frac{1}{4} (\partial_\mu A'_\nu)^2 , \qquad + i \frac{g'}{2} (\partial_\mu \phi^\dagger \cdot \phi - \phi^\dagger \partial_\mu \phi) A_\mu$$
$$+ g'^2 |\phi|^2 A'^2_\mu$$

$L_{fermion}$ $\quad \bar{\psi} \gamma^\mu D_\mu \psi$

$$= \psi^\dagger \gamma^\mu \partial_\mu \psi - i \frac{g}{2} \bar{\psi} \gamma^\mu \tau \psi \vec{A}_\mu$$
$$- i \frac{g'}{2} \bar{\psi} \gamma^\mu \psi A'_\mu$$

mass terms

From $\langle \gamma_\mu \quad \gamma_\nu \rangle \quad \sim g_{\mu\nu}$

fermion $\quad tr\, \gamma_\mu (\gamma \cdot p' + m) \gamma_\nu (\gamma \cdot p + m) \quad g^{\mu\nu} \quad$ (single flavor)

$$= tr\,(-2\gamma \cdot p' + 4m)(\gamma \cdot p + m)$$

$$= -(2\, p \cdot p' + 4m^2) \times 4$$

$$\Rightarrow \quad \langle \gamma_\mu \quad \gamma_\nu \rangle = 4 \cdot \left(-\tfrac{1}{2}\, p \cdot p' + \cancel{4} m^2\right) g_{\mu\nu}$$

$k=0$: $\qquad (-\tfrac{1}{2} p^2 + m^2) / (p^2 - m^2)^2$

$$\rightarrow -\frac{1}{2} \frac{1}{p^2 - m^2} + \frac{1}{2} \frac{m^2}{(p^2 - m^2)^2}$$

$$\rightarrow -\frac{1}{2}\left(\Lambda^2 - m^2 \ln \frac{\Lambda^2}{m^2}\right) - \frac{1}{2} m^2 \ln \frac{\Lambda^2}{m^2}$$

$k \neq 0$ $\qquad -\frac{1}{2}\left(\Lambda^2 + \left(\frac{k^2}{2} - 2m^2\right) \ln \frac{\Lambda^2}{m^2}\right) \mp \frac{m^2}{\cancel{2}} \ln \frac{\Lambda^2}{m^2}$

So the ln terms cancel!

Lef-handed currents only:

$$\gamma_\mu \to \gamma_\mu(1+\gamma_5)/2$$

$\Rightarrow$ mass term drops out.

$$tr\left(\gamma_\mu \frac{1+\gamma_5}{2} \quad \gamma_\nu \frac{1-\gamma_5}{2}\right) \to -\frac{1}{4} p\cdot p' g_{\mu\nu} \times 4$$ $\to$ trace for each flavor

$$\Rightarrow -\cancel{4}\left(\Lambda^2 + \left(\frac{k^2}{2} - \underbrace{2m^2}_{\to\, m_1^2+m_2^2\, *}\right) \ln\frac{\Lambda^2}{m^2}\right) \times g_{\mu\nu}$$ see p. 53.

numerical factor $\frac{1}{16\pi^4}\cdot\pi^2 = \frac{1}{16\pi^2}$

3 colors ~~×2 fl.~~

mass term $m_V^2 = \frac{1}{16\pi^2}\cdot \frac{2}{\cancel{4}} m_f^2 \ln\frac{\Lambda^2}{m_f^2} \times g^2 \times \cancel{6}3$

Λ^2 term $-\frac{1}{16\pi^2}\Lambda^2 \times g^2$ per 2-compt fermion.

g: charge of fermion

Yukawa coupling

$$\langle 1 \quad 1\rangle =: -4\left(\Lambda^2 + \left(\frac{k^2}{2} - 3m^2\right)\ln\frac{\Lambda^2}{m^2}\right)$$

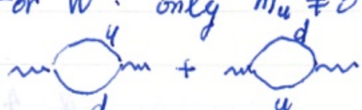
$$\frac{1}{G^2} = \frac{1}{16\pi^2}\cdot 2\ln\frac{\Lambda^2}{m^2}$$

So $m_V^2 = \frac{m_f^2}{G^2} g^2 = g^2 v^2$ $\quad g$: charge of 2-compt fermion

$$v^2 = m_f^2/G^2$$

N.B. 1) No mass generation (m_V) ~~unless ψL or ψR only~~ for pure vector.

* 2) For $W^\pm$: only $m_u \neq 0$, but two diagrams



Yukawa 

only.

What about finite terms for vector interaction ?

Evaluate $\left(-\frac{1}{2}p^2 + m^2\right)/(p^2 - m^2)^2 = -\frac{1}{2}\left(\frac{1}{p^2 - m^2} - \frac{m^2}{(p^2-m^2)^2}\right)$

$$\to \quad \frac{1}{E} + \frac{m^2}{2}\frac{1}{E^3}$$

$$\int d^3p = \frac{1}{2}\left[\Lambda\sqrt{\Lambda^2 + m^2} - \frac{m^2\Lambda}{\sqrt{\Lambda^2+m^2}}\right]$$

$$\to \frac{1}{2}\left[\Lambda^2 + \frac{m^2}{2} - m^2\right] = \frac{1}{2}\left[\Lambda^2 - \frac{m^2}{2}\right]$$

Finite part $= -m^2/4 \times \frac{-1}{16\pi^2}\cdot 4\pi^2 \cdot 4$ ← trace γ

$$\to m^2 \frac{1}{4\pi^2}$$

Euclidean 4-dim caluclation.

$$\int\left[\frac{p^3}{p^2+m^2}dp + \cancel{\tfrac{1}{2}}\frac{m^2}{(p^2+m^2)^2}p^3 dp\right]$$

$$\frac{1}{2}\int\left[\frac{x\,dx}{x+m^2} + \cancel{\tfrac{1}{2}}\frac{m^2}{(x+m^2)^2}x\,dx\right]$$

$$= \frac{1}{2}\int\left[1 - \frac{m^2}{x+m^2} + \cancel{\tfrac{1}{2}}\frac{m^2}{(x+m^2)} - \cancel{\tfrac{1}{2}}\frac{m^4}{(x+m^2)^2}\right]dx$$

$$= \frac{1}{2}\left[x - \cancel{\frac{m^2}{2}\ln} + \frac{m^4}{\cancel{2}}\frac{1}{x+m^2}\right]_0^{\Lambda^2}$$

$$= \frac{1}{2}\left[\Lambda^2 - m^2\right] \times -\frac{1}{16\pi^2}\cdot 2\pi^2\cdot 4$$

$\to \quad m^2\cdot\frac{1}{4\pi^2}$ This is the same as the 3-dim calculation.

$m_V{}^2 = \frac{g^2}{4\pi^2}m_f^2$ g: charge of 4-coupl fermion

Is this mass real, or should it be absorbed as a renormalization of Λ^2?

Self-energy from Higgs.

I. $\phi^\dagger \left(\frac{g}{2}\right)^2 (A_\mu \cdot \tau)^2 \phi \to \left(\frac{g}{2}\right)^2 A^i_\mu A^i_\mu \langle \phi^\dagger \mathbb{1} \phi \rangle$

$m_V^2 : \to \left(\frac{g}{2}\right)^2 \frac{1}{16\pi^4} \cdot \underset{\mathrm{Tr}\,\mathbb{1}}{\underset{\downarrow}{2}} \cdot \underset{2(\frac{1}{2}A_\mu^2)\, m_V^2}{\underset{\downarrow}{2}} \cdot \pi^2 \left[\Lambda^2 - m_H^2 \ln \frac{\Lambda^2}{m_H^2} \right]$

II. $-\left(\frac{g}{2}\right)^2 \int \frac{(p+p')_\mu (p+p')_\nu}{(p^2-m^2)(p'^2-m^2)} \times \overset{\nearrow \mathrm{Tr}\,\mathbb{1}}{2} \cancel{2}$ $\qquad k_\mu k_\nu$ terms $= 0$.

$k=0,\ p'=p : \to 4 \int \frac{p_\mu p_\nu}{(p^2-m^2)^2} \to g_{\mu\nu} \int \frac{p^2}{(p^2-m^2)^2}$

$= g_{\mu\nu} \int \left[\frac{1}{p^2-m^2} + \frac{m^2}{(p^2-m^2)^2} \right]$

(From p.53) $\quad (p+p')_\mu (p+p')_\nu \to 4 p''^2 g_{\mu\nu}$

$= g_{\mu\nu} \left[\Lambda^2 + \left(\frac{k^2}{3} - 2m^2\right) \ln \frac{\Lambda^2}{m^2} \right], \quad k^2 \to 0$

$\times \pi^2. \quad \times^{-1}/16\pi^4 \times (g/2)^2 \times 2$

I + II: $\quad \left(\frac{g}{2}\right)^2 \frac{1}{16\pi^2} \left[\frac{2}{3} \Lambda^2 \cancel{+ 2 m_H^2 \ln \frac{\Lambda^2}{m_H^2}} - k^2 \ln(\ ) \right]$

for SU(2) Higgs.

Again no m_H^2 term.

The Λ^2 term, for each SU(2) doublet of 2-compt fermion ~~or~~ and Higgs, cancel!

This seems true for any SU_n or U(1).

Log self-energies.

1. Problem: is $m_\pi = 0$?

meson self-energies

$\Lambda^2, \ln\Lambda^2$

$\ln\Lambda^2$

$$\frac{G^2}{2}(\sigma^2+\pi^2-v^2)^2 \to \frac{G^2}{2}(2v\tilde\sigma+\tilde\sigma^2+\vec\pi^2)^2$$

$$\to 4G^2v^2\frac{\tilde\sigma^2}{2} + 2G^2v\tilde\sigma^3 + 2G^2v\tilde\sigma\pi^2$$
$$+ \frac{G^2}{2}(\tilde\sigma^4 + 2\tilde\sigma^2\vec\pi^2 + (\vec\pi^2)^2)$$

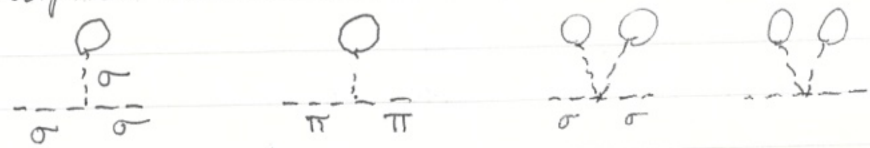
$$2G^2v\tilde\sigma^3 \to 6G^2v\tilde\sigma\langle\tilde\sigma^2\rangle$$
$$2G^2v\tilde\sigma\pi^2 \to 2G^2v\sigma\langle\vec\pi^2\rangle$$
$$\frac{G^2}{2}\tilde\sigma^4 \to \frac{G^2}{2}6\tilde\sigma^2\langle\tilde\sigma^2\rangle$$
$$G^2\sigma^2\vec\pi^2 \to G^2\langle\vec\pi^2\rangle\tilde\sigma^2 + G^2\vec\pi^2\langle\tilde\sigma^2\rangle$$
$$\frac{G^2}{2}(\vec\pi^2)^2 \to G^2\vec\pi^2(\langle\vec\pi^2\rangle + 2\langle\pi_i^2\rangle)$$

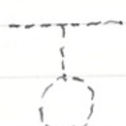
$$\left[\frac{1}{3}(2G^2v)\tilde\sigma^3 + (2G^2v)\tilde\sigma\vec\pi^2\right]^2 \to \cancel{6}\,12(2G^2v)^2\tilde\sigma^2$$

$$+ \quad 2(2G^2v)^2\vec\pi^2$$

N.B. G^2 stands for $G^2/16\pi^2$, etc

70. Tadpole contribution



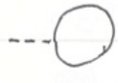
	$\pi^2/2$	$\delta\sigma^2/2$
$\cancel{\pi/\sigma}$: 	$-G^2(12\Lambda^2 - 2m_\sigma^2 \ln)$	$G^2(12\Lambda^2 - 6m_\sigma^2 \ln)$
	$-4G^2 m_\sigma^2 \ln$	$-24G^2 m_\sigma^2 \ln$
	$\underbrace{4G^2 v\langle\sigma\rangle}$ $\left(\frac{m_\sigma^2}{v}\right)$	$12G^2 v\langle\sigma\rangle$ $\left(\frac{3m_\sigma^2}{v}\right)$
$\langle\sigma\rangle$	$\frac{-v}{m_\sigma^2}\left[12G^2\Lambda^2 - 6G^2 m_\sigma^2\ln - 8N_c f^2(\Lambda^2 - m_f^2\ln)\right]$	
	$-8N_c f^2(\Lambda^2 - m_f^2\ln)$ $+\frac{k^2}{2}\ln$	$-8N_c f^2(\Lambda^2 - 3m_f^2\ln)$ $\downarrow$ $+\frac{k^2}{2}\ln$
		Set $k^2 = m_\sigma^2$ on shell.
	$2G^2\langle\sigma\rangle^2$	$6G^2\langle\sigma\rangle^2$

1. $m_\pi^2 = 0$ if $m_\sigma^2 = 4G^2 v^2$.

2. $-G^2(24\Lambda^2 + 12 m_\sigma^2 l_n) + 16 N_c f^2 \Lambda^2 = \delta m_\sigma^2$

$2(8N_c f^2 - 12G^2)\Lambda^2 = 12 G^2 m_\sigma^2 l_n$ if $\delta m_\sigma^2 = 0$.

3. $m_f = f(v + \langle\sigma\rangle) + \Sigma$ $\quad \Sigma =$ 

In $\langle\sigma\rangle$  $= 8N_c f m_f(\Lambda^2 - m_f^2 l_n)$

$= 8N_c f v(\quad) + 8N_c f \langle\sigma\rangle(\quad)$

4. To drop linear terms in σ, counter term;

$= -\frac{\lambda}{2}\phi^2 = -\frac{\lambda}{2}(\sigma^2 + 2v\sigma + \pi^2)$

$-v\lambda + f\bar\psi\psi + 2G^2 v(\cancel{8}\,3\langle\sigma^2\rangle + \langle\pi^2\rangle) = 0$

$\lambda v = -\langle\sigma\rangle m_\sigma^2$

This eliminates the tadpole from m_π^2 and replace it with $-\lambda = m_\sigma^2\langle\sigma\rangle/v$.

Since now $\delta m_\pi^2 = 0$, we compute

$\delta m_\sigma^2 - \delta m_\pi^2 = -24G^2 m_\sigma^2 l_n + 16 N_c f^2 m_f^2 l_n - 4N_c f^2 m_\sigma^2 l_n$

(If $-12G^2 + 8N_c f^2 = 0$?)

m_f is now $m_f = fv$ up to correction

Possible conditions to be imposed.

1). $\delta m_\sigma^2 = 0$ (no renormalization)

$$\to 8N_c m_f^4 - 24 N_c m_f^2 m_\sigma^2 - 3 m_\sigma^4 = 0$$

$$(m_f/m_\sigma)^2 = \frac{1}{8}\left(1 + \sqrt{1 + \frac{24}{N_c}}\right)$$

$N_c = 3 \to \; = 1/2 \qquad N_c = 1 \to 3/4$

2) Λ^2 condition $\quad 8N_c m_f^2 - 3 m_\sigma^2 = 0$

$m_f^2/m_\sigma^2 = 3/8N_c \; = 1/8 \quad N_c = 3$

$3/8 \quad N_c = 1$

$1/4 \quad N_c = 3/2$ (up only)

3) bootstrap $\langle \sigma \rangle = v$.

$\to \lambda = - m_\sigma^2$

4) no scale $\quad \lambda = -\frac{1}{2} m_\sigma^2 \quad \to \langle \sigma \rangle = v/2$

Now bring in the gauge field

$$D_\mu h^+ D_\mu h = \left(\partial_\mu h^+ + i\tfrac{g}{2} h^+ \tau\cdot A + i\tfrac{g'}{2} h^+ B\right)\left(\partial_\mu h - i\tfrac{g}{2}\tau\cdot A h - i\tfrac{g'}{2} B h\right)$$

$$= \partial_\mu h^+ \partial_\mu h + \left(\tfrac{g}{2}\right)^2 h^+ \vec{A}^2 h + \tfrac{g'}{2} B^2 h^+ h$$

$$+ 2\tfrac{g}{2}\tfrac{g'}{2} h^+ \tau_3 A B h$$

$$+ i\partial_\mu h^+ \left(\tfrac{g}{2}\tau\cdot A + \tfrac{g'}{2} B\right) h - i h^+ \left(\tfrac{g}{2}\tau\cdot A + \tfrac{g'}{2} B\right)\partial_\mu h$$

$$h = \begin{pmatrix} \sigma + i\pi_3 \\ i\pi_1 - \pi_2 \end{pmatrix} / \sqrt{2}$$

$$g A_3 + g' B = Z \bar{g} \qquad -g' A_3 + g B = \gamma \bar{g}$$

$$g / \sqrt{g^2 + g'^2} = \cos\theta \qquad g g' / \sqrt{g^2 + g'^2} = e$$

$$g' / \sqrt{} = \sin\theta \qquad g^2 / \sqrt{} = g_Z$$

$$A_3 = Z\cos\theta - \gamma \sin\theta$$

$$B = Z\sin\theta + \gamma\cos\theta$$

$$-g A_3 + g' B = \left(-Z\cos 2\theta + \gamma \sin 2\theta\right)\bar{g}$$

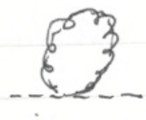
contact terms $\frac{1}{2}(\sigma^2 + \pi_3^2)\left(\frac{g_Z}{2} Z_\mu\right)^2$

$$+ \tfrac{1}{2}(\sigma^2 + \pi_3^2)\left(\tfrac{g}{2}\right)^2 (W_{1\mu}^2 + W_{2\mu}^2)$$

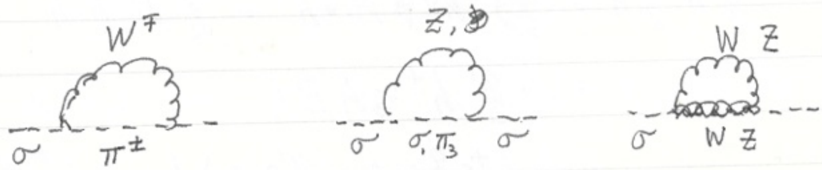
$$+ \tfrac{1}{2}(\pi_1^2 + \pi_2^2)\left(\tfrac{g}{2}\right)^2 (\quad " \quad)$$

$$+ \tfrac{1}{2}(\pi_1^2 + \pi_2^2)\left(Z_\mu \cos 2\theta - \gamma_\mu \sin 2\theta\right)^2 \bar{g}^2$$

consider first the σ self-energy



$$3\left(\frac{g}{2}\right)^2 2(\Lambda^2 - m_W^2 \ln) + 3\left(\frac{g_Z}{2}\right)^2(\Lambda^2 - m_Z^2 \ln)$$
$$\to 3\,[2m_W^2 + m_Z^2]\,\Lambda^2 - 3\,[2m_W^4 + m_Z^4]\ln$$



The second terms do not depend on mixing. so use unmixed states

$$\left(\frac{g}{2}\right)\left(-i\, h^+ A_\mu\cdot\tau\, \overleftrightarrow{\partial_\mu} h\right) - \left(\frac{g'}{2}\right)\left(i\, h^+ B_\mu \overleftrightarrow{\partial_\mu} h\right)$$

A: $3\times\left(\frac{g}{2}\right)^2 \int (p_\mu + p'_\mu)\left(g_{\mu\nu} - \frac{p'_\mu p'_\nu}{p'^2}\right)(p_\nu + p'_\nu)\,\frac{1}{p^2 p'^2}$

B: $1\times\left(\frac{g'}{2}\right)^2$ ''

$$(p+p')^2 - \frac{p'\cdot(p+p')\; p'\cdot(p+p')}{p'^2}$$

$$\cancel{= (p+p')^2 - \left(\frac{p'\cdot p}{p'^2} + 1\right)^2}$$

$$= \{p'^2[(p^2 + 2p\cdot p' + p'^2) - (p'^2 + p\cdot p')^2\}/p'^2$$

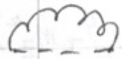
$$= [p'^2 p^2 - (p\cdot p')^2]/p'^2$$

$$\times\frac{1}{p^2 p'^2} = \frac{1}{p'^2}\left(1 - \frac{(p\cdot p')^2}{p'^2 p^2}\right)$$

$= 0$ for $k = 0$. For $p = p' + k$:

$$(2p+k)^2 - \frac{(2p+k)\cdot p}{p^2} = \frac{p^2 k^2 - (p\cdot k)^2}{p^2}$$

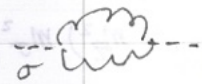
The leading Λ^2 divergence is zero. But the ln terms would depend on mixing?
Since the corrections start with k^2 (in the Landau gauge), ~~mixi~~ the coefficient of k^2 is not affected by mixing.

 $$-\int \frac{1}{(p+k)^2 \cancel{-m^2}} \frac{1}{p^2-m^2}\left[k^2 - \frac{(p\cdot k)^2}{p^2}\right]$$

$$= -\frac{3k^2}{4}\int \frac{1}{(p^2)^2} = +\frac{3k^2}{4}\ln\frac{\Lambda^2}{\mu^2}\frac{1}{16\pi^2}$$

W, Z $\to \left[\left(\frac{g}{2}\right)^2 \times 2 + \left(\frac{g_Z}{2}\right)^2\right]\frac{3}{4} m_\sigma^2 \ln$ at $k^2 = m_\sigma^2$

$\to (2m_W^2 + m_Z^2)\, m_\sigma^2 \frac{3}{4}\ln(\ )$

 $$\left(v\left(\frac{g}{2}\right)^2\right)^2 \int \frac{3}{(p^2-m^2)^2}\times 2 \to v^2\left(\frac{g}{2}\right)^4 \cdot 6 \ln(\ )$$

W, Z $\to -\left[\cancel{2}\left(\frac{g}{2}\right)^2 m_W^2 \times 2 + \left(\frac{g_Z}{2}\right)^2 m_Z^2\right]\times 6\ln(\ )$

$\to -[2m_W^4 + m_Z^4]\cdot 6\ln(\ )$

Sum of all

$$3[2m_W^2 + m_Z^2]\Lambda^2 - 3[2m_W^4 + m_Z^4]\ln$$
$$- 6[2m_W^4 + m_Z^4]\ln + \frac{3}{4}m_\sigma^2(2m_W^2 + m_Z^2)\ln$$

$$= 3[2m_W^2 + m_Z^2]\Lambda^2$$
$$- 9(2m_W^4 + m_Z^4)\ln + \frac{3}{4}m_\sigma^2(2m_W^2 + m_Z^2)\ln$$

Collecting all terms

1. Tadpoles (from old calc.)

$$\langle\sigma\rangle = \frac{v}{m_\sigma^2}\left[8N_c m_f^2 - 3m_\sigma^2 - 3(2m_W^2 + m_Z^2)\right]\Lambda^2$$

$$+ \frac{\cancel{v}\,1}{m_\sigma^2 v}\left[-8N_c m_f^4 + \frac{3}{2}m_\sigma^4 - 3(2m_W^4 + m_Z^4)\right]\ln$$

2. $$\delta m_\sigma^2 \cancel{= -8N_c m_f^2 + 3m_\sigma^2 + 3} - \delta m_\pi^2$$

$$= \left[-8N_c m_f^2 + 3m_\sigma^2 + 3(2m_W^2 + m_Z^2)\right]\Lambda^2$$

$$+ \frac{1}{v^2}\Big[16N_c m_f^4 - 6m_\sigma^4 - 4N_c m_f^2 m_\sigma^2 - 9(2m_W^4 + m_Z^4) + \frac{3}{4}(2m_W^2 + m_Z^2)m_\sigma^2\Big]\ln$$