Ginzburg - Landau form for the model of p.9.

This is equivalent to complex isotriplet $\vec{\phi}, \vec{\phi}^+$

Let $\quad \vec{\tau}\cdot\vec{\phi} = \Phi, \quad \vec{\tau}\cdot\vec{\phi}^+ = \Phi^+$

Then $\quad W = \Phi\Phi^+ - c^2$

$$\Phi\Phi^+ = \vec{\phi}\cdot\vec{\phi}^+ + i\,\vec{\tau}\cdot\vec{\phi}\times\vec{\phi}^+ \quad ?$$

solutions are of the form $\langle\vec{\phi}\rangle = (c, ic, 0)$

The relevant goldston : $\quad (0, 0, x+iy)$

Higgs $\quad (x, iy, 0) \quad$ the mass seems to be $m_H = m_f$

In the $\tau$-space ($\psi_\uparrow 1 \psi_\downarrow$ $i\psi_\uparrow \tau_3 \psi_\downarrow$, $i\psi_\uparrow \tau_1 \psi_\downarrow$)

$\sim \phi$ form a triplet.

$$\tau_2 (1, \ i\tau_3, \ i\tau_1) = (\tau_2, -\tau_1, \tau_3)$$

$$\phi \tau_2 \phi = \tau_2 (\tau_2 \phi)(\tau_2 \phi) = \tau_2 \vec{\phi}^2 = \text{invariant.}$$

$$\vec{\phi}^2 \sim 1.1 - \tau_3\tau_3 - \tau_1\tau_1 \qquad SU(2) \text{ invariant!}$$

If $\langle 1 \rangle = \langle \tau_3 \rangle = c \Rightarrow \text{mass} \sim 1 - \tau_3 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

This does not produce a self-consistent mass:

$\langle 1 \rangle$ comes from lower compt only.
$\langle \tau_3 \rangle$ " " $= -\langle 1 \rangle$

April 1, 90 reproduction of idea of Nov. 89.



$SU(2) \times U(1)_{chiral}$

$n=8$ $\quad \psi_L, \psi_R \quad SU(2)$ doublets

should be 8 ← $n=6$ $\quad \vec{a}, \vec{b} \quad SU(2)$ triplets $\quad \phi = (\vec{a} + i\vec{b}) \cdot \vec{\tau}$

$$Q_L = \sigma \cdot \partial\phi \, \psi_R + W \psi_L \qquad W = \phi^2$$

$$\phi^2 = \vec{a}^2 - \vec{b}^2 + 2i \vec{a} \cdot \vec{b}$$

$W = 0$ if $\vec{a}^2 = \vec{b}^2, \ \vec{a} \cdot \vec{b} = 0$

so take $a_x = b_y = v$

Mass term $\quad v(\tau_x 1 + i\gamma_5 \tau_y) = \pm 2, \pm 0$

broken: $\quad \tau_1, \tau_2, \tau_3 + \rho_1 \qquad$ O.K.: $\tau_3 - \rho_1, \ \rho_1 \tau_3$

massless $\quad a_z, b_z, (a_y, b_x)$ ?

Expand $W$: massive modes are $\tilde{a}_x - \tilde{b}_y, \ \tilde{a}_y + \tilde{b}_x$

# Decomposition of self-energy

$$\gamma_\mu \frac{\gamma\cdot(p-k)+m}{(p-k)^2-m^2}\gamma^\mu \frac{1}{k^2-M^2} \to \frac{-2\gamma\cdot(p-k)+4m}{(p-k)^2-m^2}\frac{1}{k^2-M^2}$$

boson on shell $\quad \delta(k^2-M^2) \to \frac{1}{2E_k} \quad E_k=\sqrt{\vec{k}^2+M^2}$

$$(p-k)^2-m^2 \to p^2-m^2+M^2-2p\cdot k \sim M^2-2p\cdot k$$

$$\sum_{k_0=\pm E_k} \frac{-2\gamma\cdot p+2\gamma\cdot k+4m}{p^2-m^2+M^2-2p\cdot k}\frac{1}{2E_k}$$

$$\to \sum \frac{-2\gamma\cdot p+2\gamma_0 k_0+4m}{M^2-2p_0k_0}\frac{1}{2E_k}$$

If $\Lambda \gg M$ : $\quad -\gamma_0\cdot\frac{1}{mE_M} \sim -\gamma_0 m\left[\frac{\Lambda^2}{m^2}-\frac{M^2}{m^2}\ln\Lambda\right]$

$$\frac{-2\gamma\cdot p+4m}{-4p_0^2E_M^2}M^2\frac{1}{E_M} \sim -\frac{M^2}{m^2}\ln\Lambda \times(-2\gamma\cdot p+4m)$$

If $\Lambda \ll M$ : $\quad \frac{-2\gamma\cdot p+4m}{M^2}\frac{1}{E_M} \sim \frac{\Lambda^2}{M^2}(-2\gamma\cdot p+4m)$

fermion on shell

$$(-2\gamma\cdot k+4m)\,\delta(k^2-m^2)\,/[(p-k)^2-M^2]$$

$$(p-k)^2-M^2 = p^2-2p\cdot k+m^2-M^2 \sim -2p\cdot k-M^2 \quad (+2m^2)$$

$$\sum_{\pm} \frac{-2\gamma_0k_0+4m}{-2p_0k_0-M^2}\frac{1}{2E_m} \qquad E_m=\sqrt{\vec{k}^2+m^2}$$

$\Lambda \gg M \quad \gamma_0\frac{1}{mE_m}+\frac{4mM^2}{4p^2E_m^2}\frac{1}{E_m} \sim \gamma_0\left[\frac{\Lambda^2}{m}+\frac{M^2}{m}\ln\Lambda\right.$ $\quad M^2 \to M^2-2m^2$

$\Lambda \ll M \quad -\frac{4m}{M^2}\frac{1}{E} \sim -4m\cdot\frac{\Lambda^2}{M^2} \quad \left.-\frac{m^2}{m}\ln\Lambda\right]$

Check. If $M \to 0$, the ln term comes from fermion-on-shell only:

$$\gamma_0 \left( \frac{\Lambda^2}{m} - m \ln \Lambda^2 \right) + (-2m) \ln \Lambda^2 \to -3m \ln \Lambda^2$$

multiply by $\cancel{4}\pi^2/(2\pi)^4$ → numerical factor $\frac{1}{8\pi^2}$

Gauge dependence: The Landau gauge has an extra term $\sim -\gamma\cdot k\,[\gamma\cdot(p-k)+m]\,\gamma\cdot k/k^2$

$$= -(\mp\gamma\cdot p + m - \gamma\cdot k) \mp \gamma\cdot k\; 2p\cdot k/k^2$$

$$= -(-\gamma\cdot p + m) \mp \gamma\cdot k\,[2p\cdot k - k^2]/k^2$$

$$= \quad '' \quad \mp \gamma\cdot k\; \underbrace{(-(p-k)^2 + m^2)}_{\to 0}/k^2$$

The $\gamma\cdot p$ term cancels the original one.
The $m$ term changes $4m \to 3m$.

More clean separation

$$\frac{1}{k^2-M^2} \to -\frac{1}{M^2} + \frac{k^2}{M^2(k^2-M^2)}$$

1) The first term gives the ~~tadpole~~ 4-fermion type:

$$\frac{-2\gamma\cdot(p-k)+4m}{(p-k)^2-m^2}\,\frac{-1}{M^2} \to \frac{-4m}{M^2}\,\frac{1}{k^2-m^2}$$

In this case, the factor 4 came from tr. 

2) The second term: let $k^2/[(p-k)^2-m^2] \sim 1$

Then $$\frac{[-2\gamma\cdot(p-k)+4m]}{M^2(k^2-M^2)} \sim$$ 

The $4m$-term is equal & opposite to the 1st.

In the Landau gauge, $4m \to 3m$, and the rest $\to 0$.

Similar decomposition for scalar & ps.

The ~~denomin~~ numerator $= \gamma\cdot(p-k) \pm m \begin{cases} s \\ ps \end{cases}$

$$\frac{\gamma\cdot(p-k)+m}{(p-k)^2 - m^2}\,\frac{-1}{M^2} \to -\frac{1}{M^2}\,\frac{1}{4}\,\mathrm{tr}\,\frac{\gamma\cdot k+m}{k^2-m^2}$$

$$\frac{\gamma\cdot(p-k)+m}{(p-k)^2 - m^2}\,\frac{k^2/M^2}{k^2-M^2} \to \frac{1}{M^2}\left(\frac{1}{2}\gamma\cdot p+m\right)\frac{1}{k^2-M^2}$$

The quadratic terms must cancel, by construction.
The $\frac{1}{2}\gamma\cdot p$ term looks spurious, coming from $\delta((p-k)^2 - m^2)$

General strategy.

1. Decompose $\frac{1}{k^2-m_f^2}\,\frac{1}{k^2-m_b^2}$ in 

$$= \frac{1}{m_f^2-m_b^2}\left(\frac{1}{k^2-m_f^2}-\frac{1}{k^2-m_b^2}\right)$$

2. Evaluet $\Sigma$ at $p=0$

$\Rightarrow$ the kinetic term $\sim \gamma\cdot p = 0$

only mass term contributes.

N.B. gauge dependent!

3. $\Rightarrow$ reinterpret the $\Lambda^2$ terms:

=  + 

The fermion loop contr.

$$\frac{3g^2}{\underset{\substack{\parallel \\ m_V^2}}{m_b^2}-m_f^2}\left(\Lambda^2-m_f^2\ln\Lambda\right) \text{ in Landau gauge}$$

The boson loop:

$$-\frac{3g^2}{m_V^2-m_f^2}\left(\Lambda^2-m_V^2\ln\Lambda\right)$$

In general $(4-\alpha)g^2$ $\qquad \alpha=0$ Feynman

$\qquad\qquad\qquad\qquad\quad \alpha=1$ Landau

Consider purely scalar : $(\sigma, \pi)$

I. 

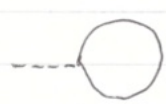
$\sigma$ : $\frac{-f^2}{m_\sigma^2-m_f^2}(\Lambda^2 - m_f^2 \ln\Lambda) \qquad + \frac{f^2}{m_\sigma^2-m_f^2}(\Lambda^2 - m_\sigma^2 \ln\Lambda)$

$\pi$ : $\frac{f^2}{\underbrace{m_\pi^2}_{0}-m_f^2}(\Lambda^2 - m_f^2 \ln\Lambda) \qquad - \frac{f^2}{\underbrace{m_\pi^2}_{0}-m_f^2}(\Lambda^2 - \underbrace{m_\pi^2}_{0} \ln\Lambda)$

II. tadpoles 

$-\frac{8G^2}{m_\sigma^2}\Lambda^2$ or $-\frac{6G^2}{m_\sigma^2}(\Lambda^2 - m_\sigma^2 \ln)$

$-\frac{2G^2}{m_\sigma^2}(\Lambda^2 \qquad )$



$+\frac{4f^2}{m_\sigma^2}(\Lambda^2 - m_f^2 \ln\Lambda)$

Tadpole cancellation :

$m_\sigma^2 = 2m_f^2$

Then I: fermi loop $f^2\left(\frac{-1}{m_\sigma^2-m_f^2} + \frac{1}{-m_f^2}\right) = -\frac{2f^2}{m_f^2} = -\frac{2}{v^2}$

II: " " $+ 2/v^2$

So the fermion & boson loops cancel separately!

Other models:

a. Nonchiral $\sigma$ only $\rightarrow$ tadpole condition

$$6G^2 = 4f^2 \rightarrow m_\sigma^2 = \frac{8}{3} m_f^2$$

I fermion loop $\quad \frac{-f^2}{m_\sigma^2 - m_f^2} = \frac{-f^2}{\frac{5}{3} m_f^2} = -\frac{3}{5v^2}$

II " " $\quad \frac{4f^2}{m_\sigma^2} = \frac{4f^2}{\frac{8}{3} m_f^2} = \frac{3}{2v^2}$

do not cancel.

b. $SU(2) \times SU(2)$ : $(\sigma, \vec{\pi})$ $\qquad m_u = m_d$ degeneracy assumed.

tadpole condition $12G^2 = \cancel{4}8f^2 \qquad m_\sigma^2 = \frac{\cancel{4}8}{3} m_f^2$

I: fermi loop $\quad \frac{-f^2}{m_\sigma^2 - m_f^2} + \frac{3f^2}{-m_f^2} = \frac{-3f^2}{m_f^2} - \frac{3f^2}{m_f^2} = -\frac{6}{v^2}$

II: fermi loop $\quad \frac{8f^2}{m_\sigma^2} = \frac{8f^2}{\frac{4}{3} m_f^2} = \frac{6f^2}{m_f^2} = \frac{6}{v^2}$

These also cancel!

c. With 3 colors, this ~~do~~ cancellation does not happen:

tadple cond. $\quad 12 G^2 = 24 f^2 \rightarrow G^2 = 2f^2$

$$m_\sigma^2 = \underset{8}{\cancel{2}}\, m_f^2 \qquad m_u = m_d$$

I: fermi loop $\quad \dfrac{-f^2}{m_\sigma^2 - m_f^2} + \dfrac{3f^2}{-m_f^2} = -4/v^2$

II: $\quad \dfrac{24 f^2}{m_\sigma^2} = 6/v^2$

In order to break $m_u \neq m_d$, one must break $SU(2) \times SU(2)$ from the beginning?