Rarita - Schwinger field

$$\partial_\mu \psi^\mu = 0 \quad , \qquad \gamma^\mu \psi_\mu = 0$$

$$\gamma \cdot \partial\, \psi_\mu = 0 \qquad \text{or} \qquad (\gamma \cdot \partial + im)\, \psi_\mu = 0$$

Modified form

$$\gamma^5 \varepsilon^{\mu\nu\lambda\rho} \gamma_\nu \partial_\lambda \psi_\rho + i m\, \psi^\mu = 0$$

$$\Rightarrow \qquad \partial_\mu \psi^\mu = 0$$

$$\gamma_\mu \gamma^5 \varepsilon^{\mu\nu\lambda\rho} \gamma_\nu \partial_\lambda \psi_\rho + i m\, \gamma_\mu \psi^\mu = 0$$

$$\Rightarrow - \gamma^5 \varepsilon^{\mu\nu\lambda\rho} \gamma_\mu \gamma_\nu \partial_\lambda \psi_\rho + i m\, \gamma_\mu \psi^\mu = 0$$

$$\Rightarrow + i \gamma^{[\lambda} \gamma^{\rho]} \partial_\lambda \psi_\rho + i m\, \gamma_\mu \psi^\mu = 0$$

if $\quad \gamma^5 = i \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho \varepsilon_{\mu\nu\lambda\rho} / 4!$

$$\gamma^{[\lambda} \gamma^{\rho]} \partial_\lambda \psi_\rho = 2 \gamma^\lambda \gamma^\rho \partial_\lambda \psi_\rho - 2 g^{\lambda\rho} \partial_\lambda \psi_\rho$$
$$= 2 (\gamma \cdot \partial)(\gamma \cdot \psi) - 2 \underset{0}{\underset{\shortparallel}{\partial^\rho \psi_\rho}}$$

$$\Rightarrow \left(-i \gamma \cdot \partial + \frac{im}{2}\right)(\gamma \cdot \psi) = 0$$

or $\quad \gamma^\rho \left(i \gamma \cdot \partial + \frac{im}{2}\right) \psi_\rho = 0$

Change the def:

$$\gamma^5 \varepsilon^{\mu\nu\lambda\rho} \gamma_\nu \partial_\lambda \psi_\rho - m \psi^\mu = 0$$

$$\Rightarrow \quad \gamma^\lambda \left(\gamma\cdot\partial + i\frac{m}{2} \right) \psi_\lambda = \left(-\gamma\cdot\partial + i\frac{m}{2} \right) \gamma\cdot\psi = 0$$

One may impose $\left(\gamma\cdot\partial + i\frac{m}{2} \right) \psi = 0$?

Then $\left(-\gamma\cdot\partial + i\frac{m}{2} \right) \gamma\cdot\psi = 0$

Impose first $(\gamma\cdot\psi) = 0$.

In the rest frame : $\gamma_i \psi_i = 0$.

$$\left(-\gamma^0 \partial_0 + i\frac{m}{2} \right) (\gamma_i \psi_i) = 0$$

How about Individual compts ?

N.B. $\varepsilon^{0123}\varepsilon_{0123} = -1$

Iterated form

$$\psi^\mu = \frac{1}{m}\gamma^5 \varepsilon^{\mu\nu\lambda\rho}\gamma_\nu \partial_\lambda \psi_\rho$$

$$= -\frac{1}{m^2}\gamma^5 \varepsilon^{\mu\nu\lambda\rho}\gamma_\nu \partial_\lambda \gamma^5 \varepsilon^{\rho\tau\sigma\eta}\gamma_\tau \partial_\sigma \psi_\eta$$

$$= +\frac{1}{m^2}\varepsilon^{\mu\nu\lambda\rho}\varepsilon_{\rho\tau\sigma\eta}\gamma_\nu \gamma^\tau \partial_\lambda \partial^\sigma \psi^\eta$$

$$= -\frac{1}{m^2}\delta^{\mu\nu\lambda}_{\tau\sigma\eta}\gamma_\nu \gamma^\tau \partial_\lambda \partial^\sigma \psi^\eta$$

$$= -\frac{1}{m^2}\delta^\mu{}_\tau \left[(\gamma_\nu \partial^\nu)\gamma^\tau(\partial\cdot\psi) - \gamma_\nu\gamma^\tau \Box \psi^\nu\right]$$

$$- \frac{1}{m^2}\delta^\mu{}_\sigma \partial^\sigma\left[\gamma_\nu(\gamma\cdot\partial)\psi^\nu - \gamma_\nu\gamma^\nu(\partial\cdot\psi)\right]$$

$$- \frac{1}{m^2}\delta^\mu{}_\eta\left[(\gamma_\nu\gamma^\nu)\Box\psi^\eta - (\gamma_\nu\partial^\nu)(\gamma\cdot\partial)\psi^\eta\right]$$

$$= -\frac{1}{m^2}\Big[-\gamma_\nu\gamma^\mu\Box\psi^\nu$$
$$+ \partial^\mu \gamma_\nu(\gamma\cdot\partial)\psi^\nu$$
$$+ (4-1)\Box\psi^\mu\Big]$$

$$= -\frac{1}{m^2}\Big[\Box\gamma^\mu(\gamma\cdot\psi) - 2\Box\psi^\mu$$
$$- \partial^\mu(\gamma\cdot\partial)(\gamma\cdot\psi)$$
$$+ 3\Box\psi^\mu\Big]$$

$$= -\frac{1}{m^2}\left[\Box\psi^\mu + (\gamma^\mu\Box - \partial^\mu\gamma\cdot\partial)(\gamma\cdot\psi)\right]$$

$$= -\frac{1}{m^2}\left[\Box\psi^\mu + (\gamma^\mu\gamma\cdot\partial - \partial^\mu)(\gamma\cdot\partial)(\gamma\cdot\psi)\right]$$

$$= -\frac{1}{m^2}\Box\psi^\mu - \frac{1}{2m^2}[\gamma^\mu,\gamma^\nu]\partial_\nu(\gamma\cdot\partial)(\gamma\cdot\psi)$$

From this:

$$\gamma\cdot\psi = -\frac{1}{m^2}\Box(\gamma\cdot\psi) \mp \frac{3}{m^2}\Box(\gamma\cdot\psi) = \frac{-4}{m^2}\Box$$

Spin $3/2$ mass $= m$

$1/2$ $m/2$

Sign of energy:

In the rest frame, $\psi_0 = 0$.

$$\psi^i = \gamma^5 \varepsilon^{ijok}\gamma_j\dot{\psi}_k \qquad \gamma_j = i\rho_2\sigma_j \qquad \gamma^5 = \rho_1$$

$$= i\rho_1\rho_2\,\varepsilon^{ijk}\sigma_j\dot{\psi}_k$$

$$= -\rho_3\,\varepsilon^{ijk}\sigma_j\dot{\psi}_k$$

$$(L^i)^{jk} = -i\varepsilon^{ijk} = -(L^j)^{ik}$$

$$\psi^i = i\rho_3(\vec{L}\cdot\vec{\sigma})^{ik}\dot{\psi}_k = i\rho_3(L\cdot\sigma)\dot{\psi}$$

$$L\cdot\sigma = (\tfrac{1}{2}\sigma+L)^2 - \tfrac{3}{4} - 2 = \begin{cases} \tfrac{3}{2}\cdot\tfrac{5}{2} - \tfrac{11}{4} = 1 & j = 3/2 \\ \tfrac{1}{2}\cdot\tfrac{3}{2} - \tfrac{11}{4} = -2 & j = 1/2 \end{cases}$$

$$\Rightarrow \quad m_{3/2} = -2m_{1/2}, \quad \mathrm{tr}\, m^{-1} = 0.$$

Various symmetry breaking schemes

1. $H_{int} \sim (\bar\psi\psi)^2 + (\bar\psi\vec\tau\psi)^2 \sim 1^2 + \tau\cdot\tau$

$SU(2) \times U(1) = U(2)$

ψ may be Majorana.

a) $\langle\bar\psi\psi\rangle = \langle\bar\psi\tau_3\psi\rangle = c$

$\Rightarrow \quad m = (1+\tau_3)\, c = 2c,\ 0$

gap eq. $\quad m = 1\, \overset{\psi_1}{\bigcirc} + \tau_3\, \overset{\psi_1}{\bigcirc} = 2m\, I(\Lambda, m)$

$\Rightarrow \quad 1 = 2I(\Lambda, \underset{2c}{\underset{\|}{m}})$

b) $\langle\bar\psi\psi\rangle = c', \quad m = c'$

$m = 1\, \bigcirc^{\psi_1, \psi_2} = 2c'\, I(\Lambda, m)$

$\Rightarrow \quad 1 = 2I(\Lambda, \underset{c'}{\underset{\|}{m}})$

c) $\langle\bar\psi\tau_3\psi\rangle = c', \quad m_1 = c', \quad m_2 = -c'$

$m_1 = \tau_3 \bigcirc = c' I_1 - (-c') I_2 = 2c'\, I(\Lambda, \underset{c'}{\underset{\|}{|m|}})$

$m_2 = -\tau_3 \bigcirc = -2c'\, I(\Lambda, |m|)$

$\Rightarrow \quad 1 = 2\, I(\Lambda, \underset{c'}{\underset{\|}{|m|}})$

So there solutions are degenerate in the lowest order.

Symmetries broken: $\quad$ a), c) $\quad SU(2) \to \tau_\perp$ goldstons.

b) none.

2. $1\cdot 1 - \gamma_5\gamma_5 + \tau\cdot\tau - \gamma_5\gamma_5\,\tau\cdot\tau \qquad U(2)_L \times U(2)_R$

sol. b) $\rightarrow [U(1)\times SU(2)]_{L+R}$

goldstons $i\gamma_5,\ i\gamma_5\vec{\tau}$

sol a) $\rightarrow (1,\ \tau_3)$

goldstons $\tau_\perp,\ i\gamma_5(1+\tau_3),\ i\gamma_5\tau_\perp$

sol c) $(1,\ \tau_3,\ \gamma_5\tau_\perp)$

goldstons $i\gamma_5\tau_3,\ \tau_\perp,\ i\gamma_5$

3. $1\cdot 1 + \tau_3\cdot\tau_3 - \gamma_5\gamma_5\,\tau_\perp\cdot\tau_\perp$

group $= (\tau_3,\ \gamma_5\tau_\perp)$ $SU(2)$

$(1,\ i\gamma_5\tau_\perp)$ ~~doublet~~ triplet.

$\langle 1\rangle \neq 0 \rightarrow$ goldstons $\gamma_5\tau_\perp$

$\langle\tau_3\rangle \neq 0$ no broken symmetry.

4. Majorana reps.

Let $\bar{\Psi} = \begin{pmatrix} \psi_1 \\ \psi_1^c \\ \psi_2 \\ \psi_2^c \end{pmatrix}$ $\begin{matrix} \}\rho \\ \}\rho \end{matrix}$ $\}\tau$ $\psi = \psi_L$ (or ψ_R)

$\psi^c = \sigma_2 \psi^+$

group: $U(1)$ $\rho_3 \times 1$

$SU(2)$ $\rho_3 \times \tau_3$, $\rho_3 \times \tau_1$, τ_2

preserves the Majorana property.

Mass terms: $(\rho_1, \rho_2\tau_1, \rho_2\tau_3)$ triplet $t^{(1)}$

$(\rho_2, \rho_1\tau_1, \rho_1\tau_3)$ " $t^{(2)}$

$\rho_1\tau_2$ singlet $s^{(1)}$

$\rho_2\tau_2$ " $s^{(2)}$

$U(1)$: $\begin{cases} t^{(1)} \longleftrightarrow t^{(2)} \\ s^{(1)} \longleftrightarrow s^{(2)} \end{cases}$

$H_{int} \sim \vec{t}^{(1)} \cdot \vec{t}^{(1)} + \vec{t}^{(2)} \cdot \vec{t}^{(2)}$ $(\oplus\ s^{(1)} \cdot s^{(1)})$

a) Let mass term $\sim \rho_1 + \rho_1\tau_3$ $m \neq 0$ only for ψ_1.
symmetry ~~completely~~ broken to $\rho_3(1-\tau_3)$
goldstons $\rho_1\tau_1$, $\rho_2\tau_1$, $\rho_2(1+\tau_3)$

diagonal couplings: ~~ρ_2 both ψ_1 & ψ_2~~

$\rho_2(1+\tau_3)$ ψ_1 only.

They are pseudoscalar

off-diagonal: $\rho_1\tau_1$ scalar if $P = \rho_1$
$\rho_2\tau_1$ ps (if $P = \rho_1\tau_3$ vice versa.)

ρ_3 here is equivalent to γ_5.

The group is generated by

$$(\gamma_5) \text{ and } (\tau_2, \gamma_5\tau_1, \gamma_5\tau_3)$$

$$H_{int} \sim 1 - \gamma_5\gamma_5 + (\tau_1\tau_1 + \tau_3\tau_3) - \gamma_5\gamma_5(\tau_1\tau_1 + \tau_3\tau_3)$$

$$(+ (1-\gamma_5\gamma_5)\tau_2\tau_2)$$

With the addition of the ~~seab~~ second term the group becomes $U(2)_L \times U(2)_R$ of p. 8.

b) $m = \rho_2\tau_1 + \rho_1\tau_3$

π: $\underset{\text{double}}{\rho_1\tau_1 - \rho_2\tau_3}$, ρ_2, ρ_1

H: 1, $\rho_3 - \tau_2$

c) $m = \rho_1 + \rho_1\tau_2$ requires $U(2)\times U(2)$, reduces to model 2 of p. 8.

Other breakings in $U(2)\times U(2)$

$m \sim \tau_3 + \gamma_5\tau_1$

π: τ_2, $-\tau_1+\gamma_5\tau_3$, $\gamma_5\tau_2$, 1, γ_5.
double

H: 1, $\gamma_5\tau_2$, $\tau_2-\gamma_5$

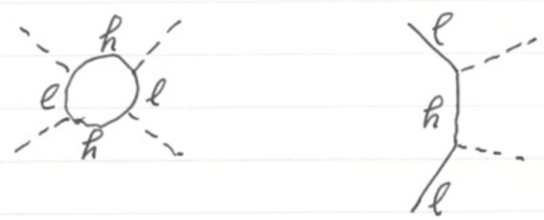
Remarks.

From the original observation re. emergence of supercurrents, the induced bosons should be those that bridge light & heavy fermion sectors. As such, they do not bridge light sectors.

$\Rightarrow$ noninteracting supersymmetry.

Q: the role of the Nicolai map?

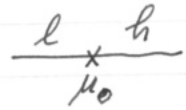
The heavy fermions mediate self-coupling of light bosons, but cannot produce Yukawa coupling to light fermions, only pair coupling.



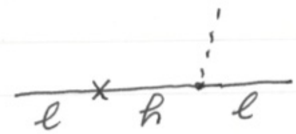
$V(\phi) \sim \phi^4$ $\qquad$ $U\bar{\psi}\psi \quad U \sim \phi^2$

They are inconsistent with SUSY.

Introduce a trigger spurion (bare off-diagonal mass)



Then one can have



The light boson also becomes massive. ?

Check on supercurrents

With interaction, $Q_{\mu R} = Q^{(1)}_{\mu R} \oplus Q^{(2)}_{\mu R}$

$$Q^{(1)}_\mu = \sigma^\lambda \partial_\lambda \phi \tilde{\sigma}_\mu \psi_R \qquad Q^{(2)}_\mu = W \sigma_\mu \psi_L$$

$$\partial_\mu Q^{(1)\mu} = \partial_\lambda \partial_\mu \phi \sigma^\lambda \tilde{\sigma}^\mu \psi_R + \partial_\lambda \phi \, \sigma^\lambda (\tilde{\sigma}^\mu \partial_\mu) \psi_R$$

$$= \Box \phi \, \psi_R + \quad ''$$

Let

$$\Box \phi = -V'(\phi) \qquad (\tilde{\sigma}\cdot\partial)\psi_R = -iU(\phi)\psi_L$$

$$\partial\cdot Q^{(1)} = -V'\psi_R - (\sigma\cdot\partial\phi)\, iU\psi_L$$

Next,

$$\partial\cdot Q^{(2)} = \partial_\mu W \sigma^\mu \psi_L + W(\sigma\cdot\partial)\psi_L$$

$$= W'(\sigma\cdot\partial\phi)\psi_L + W(\sigma\cdot\partial)\psi_L$$

$$= W'(\sigma\cdot\partial\phi)\psi_L - W i U \psi_R$$

So $\quad V' = -iWU, \quad W' = iU$

$$\rightarrow \begin{cases} V' = -WW' = -\frac{1}{2}(W^2)' & \text{if } W = \bar{W} \\ U = -iW' \quad\;\; = -(W\bar{W})' & \text{if } W = W(\phi),\ \bar{W} = W(\bar{\phi}) \end{cases}$$

Redefine $W \rightarrow +iW$ then

$$\begin{cases} V' = \frac{1}{2}(W^2)' \rightarrow V = \frac{1}{2}W^2 \ (\text{or } W\bar{W}) \\ U = W' \end{cases}$$

The Hamiltonian constructed out of $\{Q, Q^\dagger\}$

has a potential $\cancel{\tfrac{1}{2}}\, V = W^2$, $\quad H = \nabla\phi^\dagger \cdot \nabla\phi + V$

but the Yukawa coupling is $\frac{1}{2} W'$

For the real susy (Wess-Zumino),

$$V = W\bar{W}, \qquad U = W'$$

$\because$ mass term $\sim \frac{W'}{2}\psi\sigma_2\psi$ (or $\frac{\bar{W}'}{2}\psi^\dagger\sigma_2\psi^\dagger$)

Improved currents.

Consider $Q'_\mu = \sigma^\lambda \tilde{\sigma}_\mu D_\lambda \psi_R \oplus W \sigma_\mu \psi_L \oplus \tilde{\sigma}\cdot F \sigma_\mu \psi_L$

This does not cause a change in evaluating

$\{\bar{Q}_0, \bar{Q}_0^\dagger\}$

Q: In the case of $D_\lambda \to \partial_\lambda$, this procedure is equivalent to the more proper definition?

$$Q_\mu \to Q_\mu - \tfrac{1}{2}\partial_\lambda[(\sigma^\lambda\tilde{\sigma}^\mu - \sigma^\mu\tilde{\sigma}^\lambda)\phi\psi] \equiv Q''_\mu$$

$\sigma^\mu Q_\mu =: \quad \sigma^\mu\sigma^\lambda\tilde{\sigma}_\mu = -2\tilde{\sigma}^\lambda$

$\sigma^\mu \sigma_{\lambda\rho}\tilde{\sigma}_\mu = 0$

so $\sigma\cdot Q = -2(\sigma\cdot\partial\phi)\psi_R \oplus 4W\psi_L$

$= \cancel{\phi 2iU\psi_L \oplus 4W\psi_L}$

$\tilde{\sigma}^\mu \partial_\lambda(\sigma^\lambda\tilde{\sigma}_\mu - \sigma_\mu\tilde{\sigma}^\lambda) = -6\partial_\lambda\tilde{\sigma}^\lambda$

$\sigma\cdot Q'' = 2iU\psi_L \oplus 4W\psi_L$

$\quad + 3\partial\cdot\tilde{\sigma}(\phi\psi_R)$

If the shifted term is weighted by $\frac{2}{3}$, the $\sigma\cdot\partial\phi$ terms cancel.

$\sigma\cdot Q'' = 2\phi\tilde{\sigma}\cdot\partial\psi_R \oplus 4W\psi_L = -2i\phi U\psi_L + 4W\psi_L$

$= 0$ if $iU\phi = 2W$ but $U = W'$

so $W \sim \phi^2$

So the partial integration (spatial) is not quite equivalent to the addition of an antisymmetric term. The latter is only $\frac{1}{3}$ of the partially integrated term.

Ambiguity in defining the gauge field contribution to Yukawa.

$D\phi \cdot \psi$

$Q \sim \sim g \frac{\tau}{2} \cdot A \phi \psi_R$ or by integration by parts:

$-D$ acting on $\psi_R \to -\phi \frac{\tau}{2} g_R A \psi_R$

contraction with F:

1. $\frac{g}{2} \sum \tau_i^2 \phi \psi_R = \frac{3g}{2} \phi \psi_R$

2. $-\frac{\tau_i}{2} \phi \tau_i g_R \psi_R =: -\frac{g_R}{2}(3\phi_0 - i \vec{\tau} \cdot \vec{\phi}) \psi_R$

$= \frac{g_R}{2} \phi - \frac{3}{2} g_R \phi_0 \qquad \phi_0 = \frac{1}{2} \mathrm{tr}\, \phi$

This breaks $SU(2)_L$., but in fact $g_R = 0$.

The $U(1)$ part is: $g_\phi' \to -g_R'$

If $U(1)$ charges are $q_\phi' = \frac{\tau}{2}, q_R'$, then $q_L' = (q_\phi' + q_R')$

$= \frac{1}{2} \tau_3 + q_R'$

$q_R' = q_L' - \frac{1}{2} \tau_3 = \begin{pmatrix} q_L' - \frac{1}{2} \\ q_L' + \frac{1}{2} \end{pmatrix} \begin{matrix} u \\ d \end{matrix}$

~~If $q_L = 0 \to q$~~

$SU(2)$ charges are $q_{SU_2} = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}_L$ so $q_R' - q_{R\,SU2} =$

$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}_R \qquad = q_L' - q_{L\,SU2}$

q_R' is $\sim$ the electric charge.

Notes on Nicolai map

$$L = p^2\phi\bar{\phi} \cancel{+} V \qquad V = \cancel{\phi}\, W\bar{W}$$

Redefine $\phi \to \phi'$ so $p^2\phi'\bar{\phi}' = p^2\phi\bar{\phi} \cancel{+} W\bar{W}$

Square root of L:

Def: $$\begin{pmatrix} p\cdot\sigma\phi & W(\phi) \\ -\bar{W}(\bar{\phi}) & p\cdot\tilde{\sigma}\bar{\phi} \end{pmatrix} \equiv M$$

$$\begin{pmatrix} \cancel{\phi}\, p\cdot\tilde{\sigma}\bar{\phi} & -W \\ +\bar{W} & p\cdot\sigma\phi \end{pmatrix} \equiv \tilde{M} = M^{+c}$$

$$M\tilde{M} = \begin{pmatrix} (p^2\phi\bar{\phi} - W\bar{W}) & 0 \\ 0 & p^2\phi\bar{\phi} - W\bar{W} \end{pmatrix}$$

$$\frac{\partial M}{\partial \phi} \cancel{\phi} + \frac{\partial M}{\partial \bar{\phi}} = \begin{pmatrix} p\cdot\sigma & W' \\ -\bar{W}' & p\cdot\tilde{\sigma} \end{pmatrix}$$

$$\det M = \det \tilde{M}$$

Essentially $p^2 - W\bar{W} = \cdot\, \tilde{M} M$