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A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)

(In absence of the author the paper was presented by G. Jona-Lasinio.)

I. In recent years it has become fashionable to apply field-theoretical techniques to the many-body problems one encounters in solid state physics and nuclear physics. This is not surprising because in a quantized field theory there is always the possibility of pair creation (real or virtual), which is essentially a many-body problem. We are familiar with a number of close analogies between ideas and problems in elementary particle theory and the corresponding ones in solid state physics. For example, the Fermi sea of electrons in a metal is analogous to the Dirac sea of electrons in the vacuum, and we speak about electrons and holes in both cases. Some people must have thought of the meson field as something like the shielded Coulomb field. Of course, in elementary particles we have more symmetries and invariance properties than in the other, and blind analogies are often dangerous.

At any rate, we should expect a close interaction of the two branches of physics in terms of concepts and mathematical techniques, which make up the content of quantum field theory. In this talk we are going to show another possibility of such an interaction, but this time in the opposite direction to what has been the general trend. Namely, the model of elementary particles we are going to talk about is motivated by the mathematical theory of superconductivity which was first worked out with great success by Bardeen, Cooper and Schrieffer.[1] The characteristic feature of the theory is that the ground state of a superconductor is found to be separated by a gap from the excited states, which, of course, has been confirmed experimentally. The gap is caused by the fact that the attractive phonon interaction between electrons produce correlated pairs of electrons with opposite momenta near the Fermi surface, and it takes a finite amount of energy to break the correlation.

The BCS theory was given an elegant mathematical basis by Bogoliubov[2], who introduced a coherent mixture of electrons and holes to discuss the elementary excitations (quasi-particles) in a superconductor. It is easy to see that such a particle has a finite "rest energy," which corresponds to the finite energy gap. Let us assume the following equations for electrons near the top of the Fermi surface:

$$E\,\psi_{p+} = \varepsilon_p \psi_{p+} + \phi\,\psi^{+}_{-p-} \qquad (1)$$
$$E\,\psi^{+}_{-p-} = -\varepsilon_p \psi^{+}_{-p-} + \phi\,\psi_{p+} \,.$$

ψ_{p+} is the wave function for an electron of momentum p and spin + (up), and ψ^{+}_{-p-} is one for a hole of momentum p and spin +, which means the absence of an electron of momentum -p and spin - (down). ε_p is the kinetic energy measured from the Fermi surface; ϕ is a constant.

Eq. (1) gives the eigenvalues

$$E_p = \pm\sqrt{\varepsilon_p^2 + \phi^2}\,. \qquad (2)$$

So it takes an amount of energy $2|E_p| \geqslant 2\phi$ to excite such a quasi-electron from the lower to the upper state. The quantity ϕ is actually obtained as a self-consistent, self-energy (Hartree-Fock field) from the phonon-electron interaction. One finds that where $\hbar\omega$ is the mean phonon frequency, and ρ the effective electron-electron interaction energy density on the Fermi surface.

Eqs. (1) and (2) bear a striking resemblance to the Dirac equation and its eigenvalues. In the Weyl representation, the Dirac equation reads

$$\begin{aligned} E\psi_1 &= \vec{\sigma}\cdot\vec{p}\,\psi_1 + m\psi_2 \\ E\psi_2 &= -\vec{\sigma}\cdot\vec{p}\,\psi_2 + m\psi_1 \\ E_p &= \pm\sqrt{p^2+m^2} \end{aligned} \tag{3}$$

where ψ_1 and ψ_2 are the two eigenstates of the chirality operator γ_5.

This analogy may be a superficial one and devoid of physical significance. But it would also be interesting to see what would happen if we took the analogy seriously and pursued its consequences. The interpretation of Eq (3) would be then first of all that the mass of a Dirac particle is a self-energy built up by some interaction, a statement which surprises nobody. Indeed we shall find that even though the starting point looks novel, there is nothing unconventional in our model. Nevertheless, we shall also see that the analogy casts a new light on old problems, and reveals some new things which have been overlooked in the usual discussion of the self-energy problem and the symmetry properties of elementary particles.

To give an idea about our program, we draw up a list of correspondences between superconductivity and the elementary particle theory.

Superconductivity	Elementary particles
free electron	bare fermion (zero or small mass)
phonon interaction	some unknown interaction
energy gap	observed mass (nucleon)
collective excitation	meson bound nucleon pair
charge	chirality
gauge invariance	γ_5-invariance (rigorous or approximate)

As we can see from the table, our problem will be to account for the nucleons (and hyperons) and the mesons in a unified way from some basic field. There is no strong reason why we should not also consider the leptons, but for the time being we would like to exclude them. The reason will become clear later on.

As for the exact nature of the basic interaction which would produce the baryons and mesons, our model does not say what it should be. Some other guiding principles are needed for this purpose, but we do not seem to possess any convincing ones yet. So looking around for some clues, we find two possibilities rather attractive for reasons of simplicity and elegance. One is the Heisenberg type theory[3] where we consider non-linear spinor interactions. The other one is to use an analogy with the electromagnetic field. The electromagnetic field is inherently related to the conservation of charge, and the dynamics of interaction is uniquely determined by the gauge group. Attempts to generalize this idea to baryon problems have been made by Yang and Mills[4], Yang and Lee[5], Fujii[6], and recently by Sakurai[7].

Both types of theories have attractive points as well as difficulties. The most serious obstacle in any theory dealing with self-energies is the divergence problem, which is more pronounced in the Heisenberg type theory than in the other. The intermediate boson theory runs into trouble because gauge invariance requires such a field to be massless, yet massless boson fields other than electromagnetic and gravitational do not seem to exist. We do not know whether a finite observed mass can be compatible with the invariance assumption.

II. We will consider here the Heisenberg type theory because of its greater practical simplicity. The divergence will be disposed of by simple cut-off, as we do not claim to have found a way to resolve this difficulty.

Thus we adopt the following model Lagrangian for the nucleon. Isotopic spin is ignored.

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi - g[\bar{\psi}\psi\bar{\psi}\psi - \bar{\psi}\gamma_5\psi\,\bar{\psi}\gamma_5\psi] \tag{4}$$

This Lagrangian is invariant under the transformations

$$\begin{aligned} &(a) \quad \psi \to \exp[i\alpha]\psi \;;\; \bar{\psi} \to \bar{\psi}\exp[-i\alpha] \\ &(b) \quad \psi \to \exp[i\alpha\gamma_5]\psi \;;\; \bar{\psi} \to \bar{\psi}\exp[+i\alpha\gamma_5] \end{aligned} \tag{5}$$

where α is a constant c number. (Local gauge transformation is not possible here.) a) implies the nucleon number conservation; b) will be called γ_5 invariance hereafter, which implies the conservation of chirality: the number of right-handed (bare) particles minus the number of left-handed particles is conserved. We get accordingly two conserved currents

$$\frac{\partial}{\partial x_\mu}\bar{\psi}\gamma_\mu\psi = 0 \;;\; \frac{\partial}{\partial x_\mu}\bar{\psi}\gamma_5\gamma_\mu\psi = 0 \tag{6}$$

which can be directly verified.

Now we want to derive the observed nucleon mass in the Hartree-Fock approximation. Namely, we determine the mass by linearizing the interaction in which process the assumed mass is used in taking expection values. We then have the relation

$$\begin{aligned} m &= 2g\,[\langle\bar{\psi}\psi\rangle - \gamma_5\langle\bar{\psi}\gamma_5\psi\rangle] \\ &= -2g\,[\mathrm{Tr}\,S^{(m)}(0) - \gamma_5\,\mathrm{Tr}\,\gamma_5 S^{(m)}(0)] \end{aligned} \tag{7}$$

where $S^{(m)}(x)$ is the nucleon Green's function having a mass m. In momentum space this becomes

$$m = -\frac{g}{(2\pi)^3}\int\frac{m\,d^3p}{\sqrt{p^2+m^2}}\,. \tag{8}$$

A trivial solution is of course $m=0$. But with a cut-off we find also a non-trivial one

$$\frac{\pi^2}{|g|K^2} = \sqrt{1+m^2/K^2} - (m^2/K^2)\,\mathrm{sh}^{-1}|K/m| \tag{9}$$

provided that $g<0$ and $\pi^2<|g|K^2$. For $|m/K| \equiv x \ll 1$,

$$1-\pi^2/|g|K^2 \approx x^2\log(2/x), \tag{9'}$$

Eq. (8) is of the same form as the "energy gap equation" in the BCS theory. The non-analytic character of the solution with respect to the coupling constant is easily recognizable.

Our approximation scheme for the self-energy is illustrated by the following Feynman diagrams (Fig. 1).

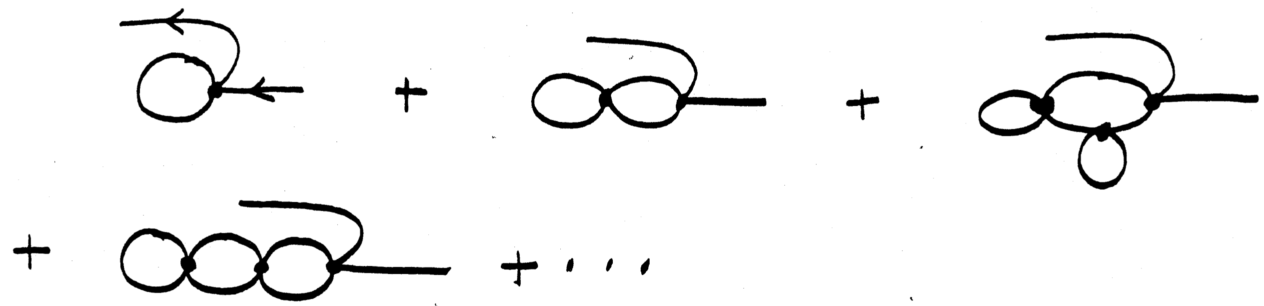

Fig. 1

Because of the attractive interaction ($g < 0$) between the virtual pair, the bubble diagrams give rise to a catastrophic change not obtained by perturbation expansion.

Thus we have created a mass out of nothing. But there are two solutions corresponding to $\mp m$, not to speak of the trivial solution $m = 0$. Presumably the vacuum corresponding to the latter solution is unstable like the normal state below the critical temperature for a superconductor. But would the two non-trivial solutions correspond to two different particles with equal mass? Heisenberg has found a similar situation in his theory. He wants to identify the two massive particles with the proton and the neutron.

Before discussing this problem, let us first worry about the conservation laws. Eq. (6) represents operator equations, so they should hold, among other things, for the matrix element between real one particle states. If we use the Dirac equation with a mass for ψ and $\bar{\psi}$, the ordinary current is all right, but the γ_5 current conservation breaks down:

$$\langle p_2 | \frac{\partial}{\partial x_\mu} \bar{\psi} \gamma_5 \gamma_\mu \psi | p_1 \rangle = -2m \langle p_2 | \bar{\psi} \gamma_5 \psi | p_1 \rangle \neq 0 . \tag{10}$$

This means that $\gamma_5 \gamma_\mu$ is not the correct vertex operator for the "dressed" particle where the mass is entirely due to the interaction. We would have to take into account the "radiative corrections" also for the vertex. The general form of the γ_5 current vertex operator $\Gamma_{5\mu}$ can be determined from Lorentz invariance and the continuity equation as follows:

$$\Gamma_{5\mu}(p_2, p_1) = \left(\gamma_5 \gamma_\mu + \frac{2im \gamma_5 q_\mu}{q^2} \right) F(q^2), \quad q = p_2 - p_1 \tag{11}$$

where $F(q^2)$ is a form factor.

On the other hand, the local field theory requires that the coefficients of $\gamma_5 \gamma_\mu$ and γ_5 obey dispersion relations of the type

$$f(q^2) = f(0) - \frac{q^2}{\pi} \int_{\kappa_0^2}^{\infty} \frac{\mathrm{Im}\, f(-\kappa^2)}{(q^2 + \kappa^2)\kappa^2} d\kappa^2 . \tag{12}$$

If $F(0) \neq 0$, then Eq. (11) has a pole at $q^2 = 0$, which means, in terms of Eq. (12), the existence of a massless, pseudoscalar, boson contributing to the form factor. A Feynman diagram showing this situation is given in Fig. 2.

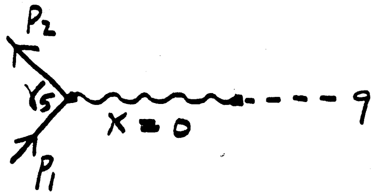

Fig. 2

Since no such boson was assumed in our theory in the beginning, we have to manufacture it somehow out of the original fermion field. A natural way would be to interpret the boson as a bound state of a fermion pair---bound because of the attractive interaction. Thus we are forced to the conclusion that if a finite mass can arise from a γ_5 invariant theory there must also be zero-mass bound states of pairs. We would nc have this situation if $F(0) = 0$, but then the total γ_5 charge

$$\langle p | \bar{\psi} \gamma_5 \gamma_4 \psi | p \rangle$$

would be zero, which leads to a contradiction (see the statement after Eq. (18)). We would not have the trouble either if the interaction were not γ_5 invariant, e.g. if the pseudoscalar term were missing in Eq. (4), which would not change Eq. (8). But we have invited the trouble deliberately because we need pseudoscalar bosons in the nucleon problem.

We can easily verify our conclusion within the present Hartree approximation. For this purpose let us set up the Bethe-Salpeter equation for the nucleon-antinucleon pair in the lowest order:

$$\begin{aligned} \Phi(p+\tfrac{q}{2}, p-\tfrac{q}{2}) = ig\Big[&\int \mathrm{Tr}\, S(p'+\tfrac{q}{2}) \Phi(p'+\tfrac{q}{2}, p'-\tfrac{q}{2}) S(p'-\tfrac{q}{2}) d^4p' \\ &- \gamma_5 \int \mathrm{Tr}\, \gamma_5 S(p'+\tfrac{q}{2}) \Phi(p'+\tfrac{q}{2}, p'-\tfrac{q}{2}) S(p'-\tfrac{q}{2}) d^4p' \Big]. \end{aligned} \tag{13}$$

$\Phi(p+\frac{q}{2}, p-\frac{q}{2})$ is the wave function for a nucleon with momentum $p+\frac{q}{2}$ and an anti-nucleon with momentum $\frac{q}{2}-p$, the total momentum being q.

We also note that if we add an inhomogenious term $\Gamma_0(p+q/2, p-\frac{q}{2})$ on the right-hand side of Eq. (13) and write Γ instead of Φ, then it would represent an integral equation for a vertex part Γ generated by Γ_0.

Now it is easy to see that the following functions satisfy the integral equation,

$$\begin{aligned} &\Gamma(p+\tfrac{q}{2}, p-\tfrac{q}{2}) = L(p+\tfrac{q}{2})\gamma_5 + \gamma_5 L(p-\tfrac{q}{2}) = -i\gamma\cdot q\gamma_5 - 2m\gamma_5, \\ &\Gamma_0(p+\tfrac{q}{2}, p-\tfrac{q}{2}) = L_0(p+\tfrac{q}{2})\gamma_5 + \gamma_5 L_0(p-\tfrac{q}{2}) = -i\gamma\cdot q\,\gamma_5, \\ &\left(L(p) = -i\gamma\cdot p - m = -i[S^{(m)}(p)]^{-1};\; L_0(p) = -i\gamma\cdot p\right). \end{aligned} \tag{14}$$

Taking $q=0$, we get $\Gamma_5=-2m\gamma_5$, $\Gamma_0=0$. In other words $\Phi=2m\gamma_5$ is a solution of the B-S equation for zero energy and momentum, which is a limiting case of a particle with zero rest mass. In fact we can construct bound state wave functions for $q_0, \vec{q}\neq 0$, $q^2=0$ starting from the above solution as the zeroth approximation.

For arbitrary q, Eq. (14) is a generalized Ward identity for the divergence of the γ_5 current $\Gamma_{5\mu}$ (Eq. (11)(F $(q^2)=1$ in our approximation.) A similar relation is known for the ordinary current[8], and can be derived in our case too. Since $L(p)|p\rangle=0$ and $\langle p|L(p)=0$, the continuity equation (10) is obviously satisfied.

Now the question of the mass degeneracy. By the γ_5 transformation $\psi\to e^{i\alpha\gamma_5}\psi$, the mass m changes into $m\exp[2i\alpha]=m(\cos 2\alpha+i\gamma_5\sin 2\alpha)$. In fact we could have assigned this form to the mass operator from the beginning. There would be no change of physical content. This arbitrariness is due to the fact that we can add any number of the zero-mass, zero-energy "mesons" to the system. The vacuum itself is degenerate in this sense.

To see this, consider the vacuum state Ω defined by

$$\psi^{(+)}\Omega=\bar{\psi}^{(+)}\Omega=0.$$

The infinitesimal γ_5 transformation is generated by

$$R=-\int\bar{\psi}\gamma_5\gamma_4\psi\,d^3x\,. \tag{15}$$

The zero-mass, zero-energy mesons are similar to the longitudinal photons encountered in quantum electrodynamics. The γ_5 and gauge transformations change the distribution of the respective quanta, but cause no physical effects. What is, then, the γ_5 quantum number for the vacuum or the one particle state? This is the eigenvalue of R, and may be written as

$$R=n_+-n_-+\bar{n}_+-\bar{n}_- \tag{16}$$

where $n_\pm(\bar{n}_\pm)$ is the number of bare fermions (anti-fermions) with $\gamma_5=\pm 1$. The nucleon number N is given by

$$N=n_++n_--\bar{n}_+-\bar{n}_- \tag{17}$$

so that $N\pm R=2(n_\pm-\bar{n}_\mp)$ is an even number.

Real nucleons with finite mass certainly are not eigenstates of R if they transform in the conventional way under R. Such a situation would be possible only if we had a complete degeneracy with respect to R. This seems to be the present case. Our world is conveniently described as a superposition of states with different R's, but there are no realizable processes which change R (a superselection rule), and hence the degeneracy does not manifest itself as physical effects.

Such an interpretation may not be quite satisfactory, but we would like to point out that a similar situation appears in the BCS theory too with respect to charge conservation. At any rate we have not found pseudoscalar photons in nature, but rather we are inclined to identify them with the mesons, which have mass in reality. Thus we will have to admit that after all nature is not γ_5 invariant. We do not know whether there is any other way out. But if the violation of the invariance were small, the foregoing results would guarantee automatically the existence of meson states, this time with a finite mass.

There are two ways to achieve our goal. One is to assume a finite bare mass m_0, which should be small compared to the observed mass m. The other is to destroy the nice symmetry of the interaction a little bit. The former looks simpler, and esthetically less objectionable. In this case, we can confirm by calculation the following results. The nucleon self-energy has now lost the freedom of γ_5 rotation, but we still get two different masses of opposite sign, in addition to the trivial perturbation solution. As m_o increases, the mass splitting grows larger, until at a certain point one of them merges with the trivial branch and disappears thereafter, leaving only the larger of the non-trivial solution.

On the other hand, the meson mass μ is proportional to $(m_o/m)^{1/2}$ so that only the solution with $m_o/m > 0$ (the largest of the three solutions)' can give rise to a stable bound pair, while the other two give rise to "ghost" mesons.

Apart from the ghost trouble, this result raises the interesting question of the possibility of producing mass multiplets of nucleons and mesons since the superselection rule does not operate any more. But the question has to be left for future study.

In our model, of course, we have obtained only neutral mesons because isospin is neglected. But the generalization is easy. For example, the interaction

$$L_{int} = -g\left[(\bar\psi\psi)^2 - \sum_{s=1}^{3} \bar\psi\gamma_5\tau_s\psi\,\bar\psi\gamma_5\tau_s\psi\right] \tag{18}$$

(where the τ's are the isotopic spin matrices) immediately leads to pseudoscalar mesons of isospin 1. The gauge group in this case consists of

$$\begin{aligned} \psi &\to \exp[i\alpha]\,\psi \\ \psi &\to \exp[i\vec\alpha\cdot\vec\tau]\,\psi \\ \psi &\to \exp[i\vec\alpha\cdot\vec\tau\gamma_5]\,\psi \end{aligned} \tag{19}$$

which correspond respectively to nucleonic charge, isotopic spin, and the γ_5x isotopic spin conservation.[9] The last two form a four-dimensional rotation group.

III. Because of some interesting features in their own right, we will discuss briefly the intermediate boson theory of primary interaction. As before, we would like to take a γ_5 invariant theory, which means that the boson is either vector or pseudovector.[9a] We can immediately write down a self-energy equation in our Hartree approximation. Namely we equate the observed mass with the familiar lowest order self-energy. Actually the self-energy consists of two parts: $\Sigma(p) = i\gamma\cdot p\,\Sigma_1(p) + \Sigma_2(p)$), Σ_1 being the wave function renormalization. Thus we get equations for Σ_1 and Σ_2 separately. These are direct analogs of the equations we encounter in superconductivity, where the boson means the phonon. It turns out that the vector interaction can give a non-trivial solution, whereas the pseudovector does not, because of the wrong sign of the self-energy. Physically speaking, the vector case causes an attractive interaction between virtual pairs, and hence a catastrophic change. Fig. 3 shows the diagrams corresponding to our approximation and the above mentioned effect.

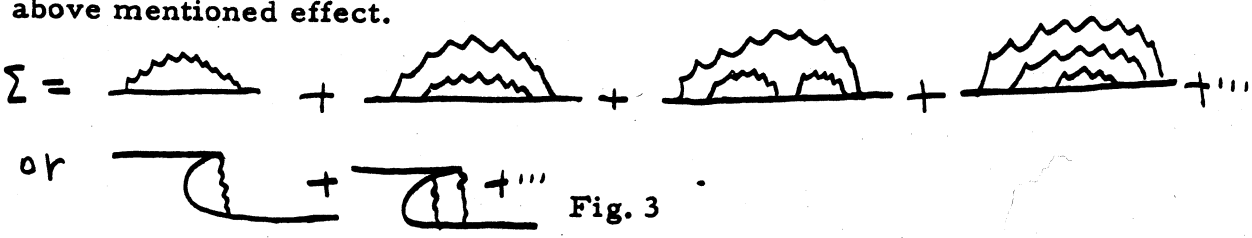

Fig. 3

It should be interesting that this is the approximation considered by Landau[10] in his discussion of the Green's functions.

The qualitative feature of the solution is similar to the one obtained in the Heisenberg type theory, except for the nature of divergences. We also obtain the zero mass bound states by solving the B-S equation in the ladder approximation. This solution was first discovered by Goldstein[11], but was considered as an abnormal object. Its raison d'etre has now become clear. According to Goldstein, however, the bound state wave function is not always normalizable. For small coupling constants, which is not really strong enough to cause such a binding, the norm of the wave function becomes negative (a ghost state).

Another intriguing question in this type of theory is whether one can produce a finite effective mass for the boson in a gauge invariant theory. Since we have done this for the fermion, it may not be impossible if some extra freedom is given to the field. Should the answer come out to be yes, the Yang-Mills-Yang-Lee-Sakurai theory of vector bosons would become very interesting indeed.

IV. We finally come to the predictions or applications of our theory. The theory essentially boils down to the compound particle model of mesons, which has found some advocates in the past (Fermi-Yang, Sakata, Okun, Heisenberg, ...). The new feature in our theory is that pseudoscalar mesons arise naturally and necessarily together with the nucleon mass as a consequence of the symmetry properties of the theory. Their type depends on the symmetry we assume. For example, it is possible to produce an isospin 1 meson but not an isospin 0 meson according to Eq. (19). There may, of course, be ordinary (or rather "accidental") bound states, but they will appear as excited or compound states of these basic mesons, like the now fashionable 2 π and 3 π resonances.

Being serious as we are about the compound particle model of the mesons, we should also try to calculate the meson-nucleon coupling constants from the basic constants. As is well known, there is no difference in the formal description of the particles whether they are elementary or compound. But if we know the wave function of a compound system, then the coupling constant is determined by it. In the case of a loosely bound system, the answer is simple. The pseudovector coupling constant is given by

$$\frac{f^2}{4\pi} = \frac{4a\mu}{m} \tag{20}$$

where m is the nucleon mass, μ the "meson" mass $= 2m - \epsilon$, and $a = \sqrt{m\epsilon}$ is the range of the wave function.

For strongly bound systems, however, the result should depend sensitively on the detailed dynamics since there are no "anomalous thresholds" in the dispersion relations for the form factors which reflect the structure of the wave function. Nevertheless let us extrapolate the above formula and see what happens. We find for the pion

$$f^2/4\pi = \frac{1}{n}\frac{4\mu}{m} \tag{21}$$

where we have also taken into account that different fermion pairs (proton, neutron, Λ, Σ, Ξ) may contribute to the pion state and n is the number of such pairs, assuming equal amount of contribution and neglecting mass fine structure. If we include all possible baryon combinations, then $n = 8$, and $f^2/4\pi = 0.075$!

We have to admit that we do not yet know what type of Lagrangian can possibly give rise to the observed baryon and meson spectrum. How much internal degree of freedom do we need to start with in order to account for the observed particles (and not to account for non-existing ones)? Why do the baryon masses split? Can we assume a high degree of symmetry properties in the beginning and yet come out with a smaller amount of apparent symmetries? The last question seems particularly relevant since we have found an example in the conflict between γ_5 invariance and finite mass. Here it should be helpful to seek analogies in solid state physics for better physical understanding.

For example, it is not hard to foresee how nature can manifest itself in an unsymmetric way while keeping the basic laws symmetric if we compare the situation with ferromagnetism. In an ordinary material, the ground state of a macroscopic body has spin zero practically, so that there is no preferred axis in space. In the ferromagnetic case, on the other hand, all the spins are parallel in the ground state, and they must point in some direction, thereby creating an assymetry in reality. Such spontaneous polarizations may be happening in the world of elementary particles too.[12]

We close this section with an application to weak interactions. So far this seems the most interesting and useful result coming out of our model.

The first question is, what is the renormalization of the vector and axial vector currents of the nucleon due to "strong" interactions, assuming there was a universal Fermi coupling in the beginning? Adopting our Heisenberg model, it can be shown that there will be no renormalization effect, and $g_V = g_A$ as long as we keep strict γ_5 invariance. The fact that g_A/g_V is only approximately unity implies then that there is a small violation of the invariance in agreement with the previous conclusion.

But this is not the whole story. We have already derived the nucleon axial vector vertex $\Gamma_{5\mu}$, which will also appear in the weak processes. The small violation of the invariance gives the meson mass μ, but will affect the form factor F of Eq. (11) relatively little. Thus we may be able to write

$$\Gamma_{5\mu}(p_2,p_1) = \gamma_5\gamma_\mu F_1(q^2) + \frac{2im\gamma_5 q_\mu}{q^2+\mu^2}F_2(q^2) \tag{22}$$

$$\approx \left(\gamma_5\gamma_\mu + \frac{2im\gamma_5 q_\mu}{q^2+\mu^2}\right)F(q^2)$$

where now $F(0) = g_A/g_V = 1.25$. The second term of Eq. (22) is small compared to the first for the actual beta decay since $q^2 << \mu^2$. For large $q^2 >> \mu^2$ we expect to recover the strict conservation: $F_1/F_2 \to 1$ as $q^2 \to \infty$.

Now we see that this second term enables one to determine the pion decay consta since, according to the dispersion theory, it represents the process going through the pion channel. Denoting the pion-nucleon (ps) coupling and the pion-lepton (pv) coupling b G_π and g_π respectively, we find

$$\sqrt{2}\, G_\pi g_\pi = 2m g_V F_2(-\mu^2) \approx 2m g_A . \tag{23}$$

Using $g_V = 10^{-5}/m^2$, $G^2/4\pi = 13.5$, we get 2.7×10^{-8} sec for the pion life time, as compared to the observed 2.56×10^{-8} sec.

Eq. (23) is exactly the same as Goldberger-Treiman[13] formula derived by an entirely different approach and rather special assumptions. But we do not think that the agreement is a coincidence. There is a certain class of models which can more or less predict this relation.[14] The essential point seems to be that the pion is effectively treated as a bound state in the G-T theory. This manifests itself through the pion renormalization constant being zero or practically zero.[15]

We can blindly generalize Eq. (23) to the strangeness changing axial vector current, where the pseudoscalar K-meson replaces the pion. Taking the ΛN vertex, for example, we again get a relation between the weak coupling g_A', $\Lambda N K$ coupling G_K and the K-lepton coupling g_K :

$$G_K g_K \approx (m_N + m_\Lambda) g_A' . \tag{24}$$

This relation does not contradict our present rather meager knowledge about these constants.

Since we are based on the compound particle model, all the considerations that have been made by various people in the past can be adopted in essence. The Gershtein-Zeldovich and Feynman-Gell-Mann idea of π -lepton vector coupling is, of course, a natural consequence of the model, though it has yet to be tested experimentally.

We feel that a systematic and quantitative calculation of the renormalization effects can be undertaken in our theory with more confidence than in the past because we have better understanding of the interrelation of different phenomena. So far we have tried to estimate the decay life times for most of the decay modes of strange particles under very crude assumptions. The result is in general satisfactory, but it is not clear as to what it really means. We have not yet understood such basic questions as the $\Delta T = \frac{1}{2}$ rule and the smallness of the hyperon beta decay rate in any fundamental way.

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DISCUSSION

Wightman: Is it true that the zero mass bound state moves up to become the π -meson when the γ_5 invariance is broken?

Jona-Lasinio: When you break the γ_5 -invariance, for example by introducing a bare nucleon mass m_0 , you dispose of an additional parameter which can be adjusted to give the desired mass.

Guth: How can you say that the model is a Heisenberg type theory if you do not specify the dynamics?

Jona-Lasinio: This theory is a Heisenberg type theory only in the sense that a non-linear spinor equation is taken as starting point. The model is then linearized self-consistantly and the cut-off which has to be introduced to eliminate divergencies should be interpreted as a dynamical effect. The mechanism responsible for it is actually unspecified.

J. Sakurai: It is important to emphasize that much of what has been reported is independent of particular models. The only things that are relevant are: a) axial-vector conservation holds; and b) the force between a nucleon and an antinucleon is attractive.

Primakoff: When you break the γ_5 invariance to introduce the π -meson mass, how do you know that $\lim_{q^2 \to 0} F(q^2) \approx g_A/g_V$?

Jona Lasinio: It is an assumption. In order to obtain both a finite π -meson mass and a renormalization of the axial vector coupling, we have to break the γ_5 invariance. So it is assumed the same γ_5 invariance violation is responsible for both effects.

A PARADOX CONCERNING POLARIZATION IN BETA DECAY

by R. H. Good, Jr. (Iowa State University, Ames, Iowa) and M. E. Rose (Oak Ridge Nat. Lab.) (Presented by M. E. Rose)

In allowed beta transitions electron polarization is $\pm$ v/c only when the observation is confined to a measurement of the electron momentum and spin. When other dynamical variables are simultaneously measured, the beta particle polarization will be 100% under circumstances to be described below. It is emphasized that when the beta particles are completely polarized the density matrix for the process corresponds to the formation of pure states (one eigenvalue is unity and the other three eigenvalues are zero).

For pure Fermi transitions the only other variable which is relevant is the neutri direction. Then, aside from smearing effects arising from a lack of perfect energy and angular resolution, pure states are always produced - that is, the beta polarization is complete. This polarization is, in fact, equal to the unit vector in the direction of the neutrino motion as seen from the reference system in which the electron is at rest. The statement made above presupposes a two-component neutrino and, consequently, one sees that if any variable is averaged over in this case, impure states (with less than 100% β -polarization) are involved.

These results for the Fermi transition are completely understandable. However when one considers Gamow-Teller transitions a rather paradoxical situation arises. Now the nuclear orientation is added as a new dynamical variable. If the nuclei are