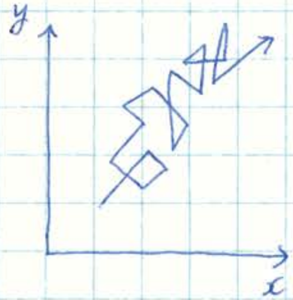
7/~~4~~5. Reduction to 1-dimensional problem

The fluctuations in x & y coordinates are like Brownian motion. To compute $\int A_i dx^i$



$$= \sum_n \vec{A}_n \cdot (\vec{x}_{n+1} - \vec{x}_n)$$

we have to take into account the topology.

Instead of Following the motion of (x,y) with t, choose x as the independent variable. Then we have pair creation & annihilation of paths:

So let $\phi_{(x)}$ be an operator $\sim a^{+}a$, ~~a^+~~ $a^{+}a^{+}$, $a\, a$

Kinetic energy.
$$\sum\left(\frac{\Delta x_n}{\Delta t}^2 + \frac{\Delta y_n}{\Delta t}^2\right) = \sum \frac{\Delta x_n}{\Delta t}^2 + \sum\left(\frac{\Delta x}{\Delta t}\right)\left(\frac{\Delta y_n}{\Delta x}\right)^2 \Delta x$$

keeping Δx, Δt fixed, and the label n is linked to x instead of t. $y(x_n)$ now is an operator.

~~Here y~~ y

To each ~~pr~~ path created, associate a propagator

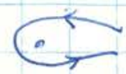
$$\exp\left[i \sum_n v \Delta x \left(\frac{\Delta y}{\Delta x}\right)_n^2 \right]$$ along the path.

The path always goes forward. Distinguish positive & negative "particles".

Bak to complex variables.

$$\exp[i\sum(k_i z_i + l_i \bar{z}_i)] \ \Pi(k_i/l_i)^{k_i}$$

is the basic fu.

1. Integrate over $\prod_i dk_i$ 

2. Maintain $\quad \Sigma k \, \Sigma l = m^2$

3. We may hold $N-1$ parameters $l_1, \ldots l_{N-1}$

   Then $l_N :\to -\sum^{N-1} l_i$ as $k_i \to \infty$

This does not require phase rotation of l's when a ~~k_i~~ z_i & $\bar{z}_i$ rotates.
Thus the factor $\Pi l_i^{k_i}$ is superfluous.

So modify:

1. Take a set of N variables k_1 or l_1; k_2 or l_2; ...

   and $\Pi \, dk \, \Pi \, dl \, \Pi . k_i^{\alpha_i} \, l_j^{-\alpha_j}$

2. Fix $N-1$ momenta $\quad k_1$ or l_1; k_2 or l_2; ..

3. Subject to $\quad \sum_{all} k \sum_{all} l = m^2$

In this case, as one $k_i \to \infty$, then $\sum^{all} l \to 0$

When a $k \to \infty$ & an $l \to \infty$: there must be at least one more k or $l \to 0$, but there will be a problem if their phases do not match.

Example: 2 pts,

$k_1, k_2 ; l_1, l_2$

Fix l_2. Then $(k_1+k_2) \sim 1/l_1$

1. $k_1, k_2 \to \infty$. l_1 finite O.K.
   $k_1+k_2 \to \infty$
2. $k_1, l_1 \to \infty$ then $k_1+k_2 \to 0$.

   so $k_2 \to \infty$ but if $\left.\begin{matrix}k_1\\k_2\end{matrix}\right)$ have domain ////////////

   then there is a problem? Both $\mathrm{Im}\, k_1, k_2 > 0$

3. As $l_1 \to \infty$, $\mathrm{Im}\, 1/l_1 \sim \mathrm{Im}\, k_1 > 0$

   $\downarrow \frac{\mathrm{Im}\,\bar{l}_1}{|l_1|^2} \sim \frac{\mathrm{Im}\, k_1}{|l_1|^2}$

   This $\to \infty$ if arg l_1 is fixed.

So we can allow $\mathrm{Im}\,(k_1+k_2) \to 0$. But the allowed common region for (k_1, k_2) becomes $\to 0$.

It seems then that we had better take a set $\{k\}$ or $\{l\}$, and not a mixture.

It can still happen that $\Sigma' k \to 0$ so that one of the l's $\to \infty$. We must avoid this by a proper choice of the paths k_i. How can this be possible?

$\Sigma' k = 0$ is a $N-1$ dim surface: k_N determined from $k_1, \cdots k_{N-1}$

In the neighborhood of $\Sigma k = 0$, we must make a detour.

If we stick in a "δ-fn" $\delta(\Sigma k \Sigma l - m^2)$,

and then integrate over l_N, say, we get an extra factor

$1/\Sigma k$

so this might be a way to ~~make~~ define an integral which is symmetric w.r. to all l's, and we have to skirt around the pole in an appropriate way.

Thus the basic integrand is

$$\exp[i(\Sigma k_i z_i + \Sigma l_i \bar{z}_i)] \, \Pi k_i^{\varkappa_i} / \Sigma k_i \quad \Pi \, dk_i$$

$$\text{with} \quad \Sigma k \, \Sigma l = m^2$$

Q: as one $z_j \to 0$, is it $< \infty$?
$\bar{z}_j$

Other variables fixed, we consider $dk_j \, k_j^{\varkappa_j} / \Sigma k$

This had better converge for large $k \to \varkappa_j < 0$.

This calls into question of the above choice $\{k\}$ or $\{l\}$ only.

Get back to the original ideas:

3. For a set $\{\varkappa_i\}$, choose $\{k_i \text{ or } l_i\}$

so that $\begin{cases} k_i & \text{if } \varkappa_i > 0 \\ l_i & \text{if } \varkappa_i < 0 \end{cases}$

and the integration is $\prod \frac{dk_i}{k_i} \prod \frac{dl_j}{l_j}$

~~f~~fixing $N-1$ other variables

& $\sum k \sum l = m^2$ as the ~~Nth~~ constraint.

Example 1. $N = 1$.

$$\int_C \exp\left[i\left(kz + \frac{m^2}{k}\bar{z}\right)\right] k^{-\varkappa-1}\, dk \quad \sim J_\varkappa(\overset{mz}{\cancel{m|z|}}) \qquad \varkappa > 0$$

C

$k \to \frac{m^2}{l}$ $\quad dk/k \to dl/l$ ~~for $\varkappa < 0$~~.

C'

$$\int_{C'} \exp\left[i\left(\frac{m^2}{l}z + l\bar{z}\right)\right] l^{\varkappa-1}\, dl$$

If we let $\varkappa \to -\varkappa$, we obtain $J_{-\varkappa}(\underset{mz}{\cancel{m|z|}})$ which

diverges like $|z|^{-\nu}$

Problem. The Nth variable, say,

$$l_N = -\sum' l_j + \frac{m^2}{\sum k_i}$$

can also $\to \infty$ when $\begin{cases} \text{some } l_j \to \infty, \\ \sum k_i \to 0. \end{cases}$

~~so~~ but there is no harm ~~if~~ since the factor $l_N^{\kappa_N}$ is missing.

except for convergence of $\exp[\sum i l_N \bar{z}_N]$

$$\sum l \bar{z} = \sum_{j=1}^{N-1} l_j \bar{z}_j - \sum^{N-1} l_j \bar{z}_N + m^2 \bar{z}_N / \sum k$$

$$= \sum{}' l_j \underbrace{(\bar{z}_j - \bar{z}_N)}_{\text{indep of } z.} + m^2 \bar{z}_N / \sum k$$

1. This shows that there is no phase factor like $\bar{z}_j^{-\kappa_j}$ created by $\int dl_j$

2. Convergence dictated by the coordinates $(\bar{z}_i - \bar{z}_N)$

3. Extra convergence problem coming from $\bar{z}_N / \sum k$ when $\sum k \to 0$

$\Rightarrow$ The choice of mixed k's & l's are meaningless.

But one ~~can~~ could make use of one variable $\bar{z}_N$ and the zero of $\sum k$.?

7/10 Previous prescriptions are no good. They stumble on the zeros of Σk.

Alternative :

$k_i \to \frac{1}{u-u_i}$ → No good, $\mathrm{Im}\,\frac{1}{u-u_i}$ not definite

u_i in the upper (or lower) half plane.

$\exp\left[i \sum \frac{1}{u_i - a_i} z_i + i\, l \bar{z} \right]$ $\qquad l = m^2 / \Sigma \frac{1}{u-u_i} \neq \infty$?

$\times \prod (u_i - a_i)^{\kappa_i} \, du_i$

Each contour around a_i is ~ 

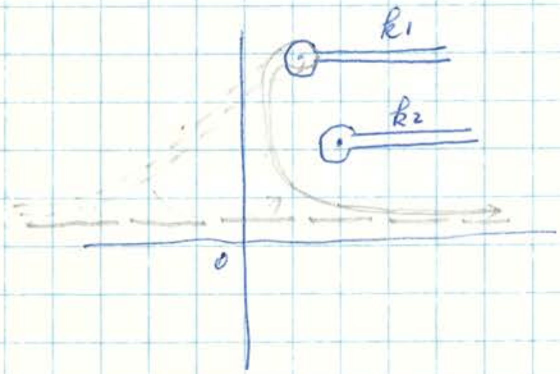
Convergence as $z_i \to 0$: $\kappa_i > -1$

However one does it, it is necessary to correlate the phases of different k's.

How about displacing $k_i^{\kappa_i} \to (k_i - a_i)^{\kappa_i}$ $\quad \mathrm{Im}\, a_i > 0$. ?

Loop around a_i :

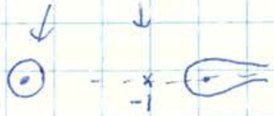
Still one gets stuck with an $\int dl$ along the real axis.



no fixed phase for k_2 even as $k_2 \to \infty$

Next $k_1 \to k_1$, $k_2 \to k_1 k_2$, $k_3 \to k_1 k_2 k_3$, etc.

$\Sigma k = k_1 (1 + k_2)$, $k_1 (1 + k_2 (1 + k_3))$ etc



Always avoid -1.

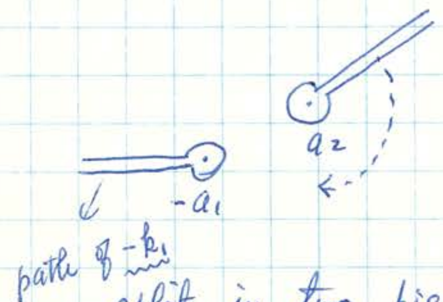
No good.

Let $k_i = \gamma_i k$ $\quad$ $\gamma_i = \rho_i e^{i\theta_i}$

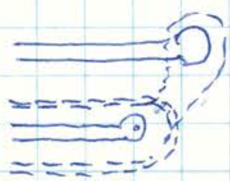
Integrate over k only? No.

Two fluxes.

$$(k_1 - a_1)^{K_1} (k_2 - \cancel{k}a_2)^{K_2}$$



These do not interfere except when k_2 rotates and crosses k_1 in which case k_2 path can be split in two pieces



The integrations around ~~a_2~~ $-k_1$

$$\int \exp\left[i \frac{\bar{z}_1 \text{ or } \bar{z}_2}{k_1 + k_2}\right] dk_2 \, (k_2 - a_2)^{K_1} \exp[i k_2 z_2]$$

$$k_2 - a_2 = k_2 + k_1 + (-k_1 - a_2)$$

This cannot be $= 0$. ~~in general~~.

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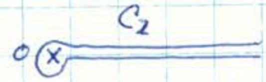
Next ansatz. k_2 integral $(0, \infty)$



x means branch pt.

k_1 integral $(0, -k_2)$

factor $k_1^{+\kappa_1} (k_2/k_1)^{\kappa_2}$

1. Loop around $\bar{z}_2$ yields a factor $e^{i\kappa_2}$ from C_2

* 2. Loop around z_1: both k_1, k_2 will rotate, so one gets $\exp(i\kappa_1)$.

The basic wave factor is

$$\exp\left[i\left(k_1 z_1 + k_2 z_2 + \frac{\bar{z}_1}{k_1+k_2}\right)\right]$$

Convergence condition: good

$z_2 \to 0 \Rightarrow \kappa_2 < -1$

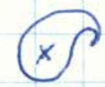
$z_1 \to 0 \Rightarrow$ for finite k_1, no cond.

As $k_1 \to \infty$ with k_2, the convergence supplied by $\exp(ik_2 z_2)$

Thus no cond. on κ_2.

This seems to have solved the problem!

* Needs a proof.

Q. Difference between  and 

is  =  around an essential singularity.

we should take  only.

Q. What about when $k_2 \to \infty$? ~~k_1 a~~ We cannot rotate k_2 without rotating z_2 as well.

Write $\quad k_1 z_1 + k_2 z_2 + \bar{z}_1 / k_1 + k_2 \qquad k_1 + k_2 = \varepsilon$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k_1 = \varepsilon - k_2$

$$= k_2(z_2 - z_1) + \varepsilon z_1 + \bar{z}_1/\varepsilon$$

So we can fix the direction of k_2 with $z_2 - z_1$

But this form cannot produce the loop around z_2.

~~Take~~ Write instead $\quad k_1 z_1 + k_2 z_2 + \bar{z}_1/(k_1 + k_2)$

$$\to \quad = k_1(z_1 - z_2) + \varepsilon z_2 + \bar{z}_1/\varepsilon$$

Integrate over ε only:

$\varepsilon \to \infty$ and $\varepsilon \to 0$ determined by z_2 & $\bar{z}_1$.

But such phase factors are not compatible?

How about $\quad \varepsilon^{\kappa_1}(\varepsilon - a)^{\kappa_2}$?

1. fixed k_1, $\int dk_2$ $(k_1+, \infty(z_2))$

2. $\int dk_1$ $(0+, \infty(z_1))$

$$\exp\left[i\left(k_1 z_1 - k_2 z_2 + \frac{\bar{z}_i}{k_1 - k_2}\right)\right] k_1^{\varkappa_1} (k_1 - k_2)^{\varkappa_2}$$

$\varkappa_1, \varkappa_2 < -1$

The order of integration important.

N.B. If 1st integrate over $k = k_2 - k_1$, then

$$k_1 z_1 - k_2 z_2 = k_1(z_1 - z_2) + (k_2 - k_1) z_2$$

so the next integration over k_1 loses the phase dependence on z_1. To avoid it, one must leave some z_1 dependence.

Thus, for example, integrate over $k_2 - \alpha(k_2) k_1$

$\alpha(k_1) \to 1$ as $k_2 \to k_1$, but $\to 0$ as $k_2 \to \infty$

E.g. $k_2 - k_1^2/k_2 = k$

Still, one cannot avoid the fact that the loop C_2 loops around k_1, so that $\exp(-ik_2 z_2) \sim \exp(-ik_1 z_2)$ and leads to the above difficulty.

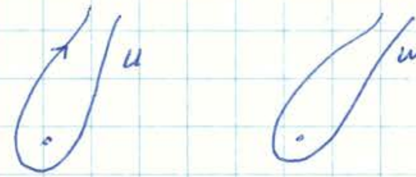
In other words, one cannot factorise the integration.

$$\begin{cases} \text{When } k_1 \approx k_2 \to \infty, \text{ it should go } \to \infty(z_1 - z_2) \text{ fixed} \\ k_1 \gg k_2 \qquad k_1 \to \infty(z_1) \\ k_2 \gg k_1 \qquad k_2 \to \infty(z_2) \end{cases}$$

~~Consider the fn as a fn of independent variables z_1, z_2.~~

~~S_0 ———— k_1~~

$$\iint \exp\left[i\left(\frac{u}{w} z_1 + \frac{w}{u} \bar{z}_2\right)\right] u^{k_1} w^{k_2} \, du \, dw$$



$u: \left(0_+, \infty(z_1/w)\right)$

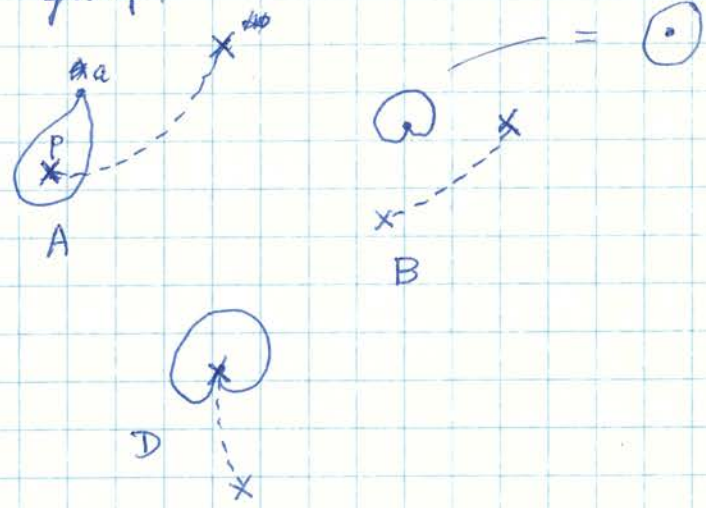
$w: \left(0_+, \infty(\bar{z}_2/u)\right)$

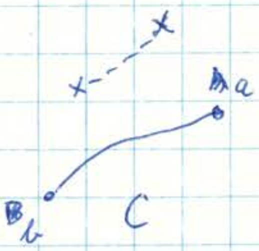
$k_1 < -1, \; k_2 < -1$

no good.

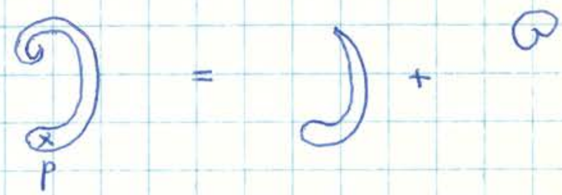
Basic monodromy group.

Line integrals





Rotate the tip around a: $\quad A \to A + B$



Rotate the whole loop around p will undo the operation

in the second sheet

or $\qquad A \to A + B \overset{\to}{=} e^{i\alpha} A$ $\quad$ diff ~~sum~~ of B's on two sheets

When $a = p$, the second operation does not change the fn.

so $\quad A \to e^{i\alpha} A$

D has the property $\quad D \to e^{i\alpha} D$

For each br. pt. one can construct a D_α. But it is inv. under other looping.

What we have here must be an Abelian group of monodromy.

Elements $na + mb$, $(n, m$ integer$)$

$\exists$ Representations $\quad O_{n,m}\, \psi^{(1)} = n\lambda\, \psi^{(1)}$

$$O_{n,m}\, \psi^{(2)} = m\mu\, \psi^{(2)}$$

What we need a repr.

$$O_{n,m}\, \psi = (n\lambda + m\mu)\, \psi$$

or $\quad \psi \sim \psi^{(1)} \otimes \psi^{(2)}$

However, we can not take tensor products of wave fns.

This is the problem. A product is not a sol. of the wave eq.

~~Fortunately,~~ If separation of variables is used,

$$\frac{\partial}{\partial z}\psi^{(1)} = ik\,\psi^{(1)}, \qquad \frac{\partial}{\partial \bar{z}}\psi^{(2)} = -il\,\psi^{(2)}$$

$$kl = m^2$$

Then $\quad \psi = \sum_k \psi_k^{(1)}(z)\, \psi_{m^2/k}^{(2)}(\bar{z}) \qquad \psi^{(1)} = \psi^{(\lambda)},\ \psi^{(2)} = \psi^{(\mu)}$

However, each $\psi_k^{(1)}$ is not a desired representation of $O_{n,m}$.

More complex case, $\quad \psi^{(1)} = \psi^{(\lambda,\mu)}$

It is possible to construct multi-valued fn $f(k)$

such that in one branch $f(k) \to \exp[ika_1]$

in another $\to \exp[ika_2]$

But how can one make use of it?

Examples

$$f_1(k) = k^{\alpha ik} \qquad f \to e^{2\pi\alpha ik} f(k)$$

$$f_2(k) = e^{i\alpha\sqrt{(k-a)(k-b)}} \qquad f \to f^{-1}$$

$$\sim e^{i\alpha k}$$

If we want $f \to e^{i(ka+c)}$

$$f_3 = f_2 \, e^{ic\sqrt{k-a/k-b}}$$

$$f_4 = f_2 \, k^{ic\sqrt{k-a/k-b}}$$

One would like to have a 2-cycle chain:

Loop (z_1) Branch I $\to$ Br. II

Br. II $\to$ Br II

No good:
not commutative.

Loop (z_2) Br I $\to$ Br I

Br II $\to$ Br I

There is an impasse:

Consider a fn $\sum_n \int \exp[i(kz+\frac{\bar{z}}{k})] f_n(k)$ over ~~an axis~~ a path and combinations thereof.

Around $z=0$: ~~k must~~ the $\int$al ~~so~~ in k reaches $\infty(z_1)$ and has a loop phase factor $e^{i\kappa_1}$. The fn must hit a singularity as $z \to a_2$, the location of the 2nd vortex.

Or $$f_n(k) \to f_n(k) \exp[i(ka_2+\frac{\bar{a}}{k})]$$

The only way to produce a singularity is for the series $\sum_n$ to diverge for some k, but the exp. factor is common to all f_n. This means that one cannot interchange Σ and $\int$. : Of course, since the paths are different for each n.

E.g. f_n has a singularity at $k=k_n$, with $k_n \to \infty$ as $n \to \infty$. Then each path ~~as~~ must grow

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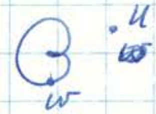
$$I = \int \exp\left[i\left(\frac{z}{w} + \bar{z}w\right)\right] \exp\left[i\left(\frac{a}{u} + \bar{u}\bar{a}\right)\right] (u-w)^{\alpha} w^{\kappa}\, dw\, du$$

$\int dw$ around 0.



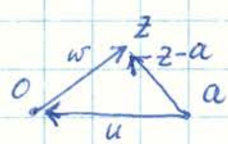
Avoid u. This gives the correct behavior near $z=0$.

Now integrate over u. Let u approach 0. There will be a pinch for w..

If phases of $\bar{z}$ & $-a$ are compatible, both w & u can take the same dip direction.

In general there will be a pinch: $w \to 0$, $u \to 0$, $|u| > |w|$ in ~~the~~ a direction other than the w dip direction.

$$\frac{z}{w} - \frac{a}{u} = \frac{z-a}{w} + a\left(\frac{1}{w} - \frac{1}{u}\right)$$



Incompatibility of u & w appears when z & a line up ~~& $|z| < |a|$~~

So assume z & a real & > 0. so w & $u > 0$.

$$\frac{z}{w} - \frac{a}{u} < 0 \quad \text{means} \quad \frac{z}{a} < \frac{w}{u} < 1$$

The crunch comes when $z/a > 1$.

The common approch of u & w should be in the direction of $z-a$.:

$\int_w$ How to get around the singularity $z = a$?

If z is not on the line $(0, a)$, there is a region of compatibility between z and a or $z - a$. So both u & w can go to zero in the same direction.

<u>I think this is the final answer.</u>

$$\int_{0_+} e^{i\left(\frac{z}{w} + \bar{z}w\right)} \, dw \, w^{+\nu-1} = 2\pi i \left(\bar{z}/z\right)^{\nu/2} J_\nu(2|z|)$$

$$\text{so} \quad I = \iint \exp\left[i\left(\frac{z}{w} + \bar{z}w\right)\right] \exp\left[-i\left(\frac{a}{u} + \bar{a}u\right)\right] \sum \left(\frac{-w}{u}\right)^n \binom{\alpha}{n} u^\alpha w^k$$

$$= \sum_{n=0}^{\infty} J_{+n+k+1}(2|z|) \left(\frac{\bar{z}}{z}\right)^{\frac{+n+k+1}{2}} J_{-n+\alpha+1}(2|a|) \left(\frac{\bar{a}}{a}\right)^{\frac{-n+\alpha+1}{2}} (-1)^n \binom{\alpha}{n}$$

More precise def. of the contours.

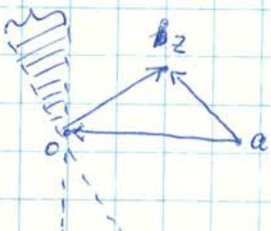
1. If z and $-a$ compatible : Choose the same dip direction.

$|k| < |l|$ for same phase angle

Take a family of similar contours. For fixed l, integrate over k with a smaller scale. As l moves over the contour, the k contour need not move. Thus the k & l contours are both fixed.

As z moves, and cross the line of incompatibility, the contours are still OK if $|z| < |a|$. ?

R = common range for k & l $\rightarrow$ This region $\neq 0$ as long as



z does not lie on line $(0, a, \infty)$

R also compatible with $z-a$ & as z moves along a radial from 0.

Let's take ∞ as the center. The exp. factor then is

$\exp[i(kz - la)]$

k loop around 0, + paths to ∞
l " " " "

One can assume the k loop > l loop.

Convergence cond. $\mathrm{Im}(kz - la) > 0$

This for any $|k|, |l| \to \infty$ with $|k| > |l|$.

So, ~~on~~ in particular, with $l = 0$, $\mathrm{Im}\, kz > 0$. (1)

But also with $|l| \to |k|$, $\mathrm{Im}(\hat{k}z - \hat{l}a) > 0$ ~~$\to 0$~~

The actual pinch happens as $l \to k$ so $\mathrm{Im}\, k(z-a) > 0$ (2)

Thus (1) & (2) are the conditions

Convergence as $z = 0$ resp. $z = a$.

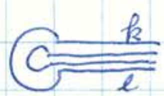
First $z = 0$ ~~$\to \int^{\infty} k^{\kappa}\, dk < \infty \qquad \kappa < -1$~~

~~$z \to a$~~: $\int^{\infty} k^{\kappa} (k-l)^{\alpha}\, dk < \infty \qquad \kappa + \alpha < -1$?

The l-integration is OK because of $\exp[i \frac{al}{k}]$

If $z = a$: $\int \exp[ia(k-l)]\, k^{\kappa} (k-l)^{\alpha}\, dk\, dl$

$\int dk$ O.K. Direction of l now arbitrary so take it to be the same as l?



This is no good. An exceptional case. k & $-l$ may be in the same direction.

Then $|k| > |l|$, and the min. distance may be arbitrary.

So the $\int$al converges always.

Multi-vortex problem.

If they are located at $\{a_n\}$ $n = 1, \cdots N$
$\{\varkappa_n\}$

$$\text{Exp}: \quad i\,[k_1(z-a_1) - \sum_{m=2}^{N} k_m a_m] + \frac{\bar{z}-\bar{a}_1}{k_1} - \sum \frac{\bar{a}_m}{k_m}]$$

$$\times \quad \cancel{k_1^{\varkappa_1} \prod_m (k_1 - k_m)^{\varkappa_m - \varkappa_1}} \text{ something.}$$

$|k_1| > |k_2| > \cdots$

The wave factor can be written

$$(k_1 - k_2)(z - a_1) + k_2(z - a_1 - a_2) - k_3 a_3 - \cdots$$

$$= \quad " \quad + (k_2 - k_3)(z - a_1 - a_2) + k_3(z - a_1 - a_2 - a_3) - k_4 a_4 - \cdots$$

So phases of k_1 controlled by $z - a_1$
k_2 $z - a_1 - a_2$
$- - -$

Loop around $z - a_1$ rotates k_1, $k_1 - k_2$
$z - a_2$ " k_2, $k_2 - k_3$
$- - - -$

Now if the factors are $\prod k_{0i}^{\varkappa_i}$, The k integrals all factor. ~~Each integral~~ The $\int dk_1$ is indep of ~~the~~ k_2 ~~loop size~~, so the restriction $|k_1| > |k_2|$ does not correlate $\int dk_1$ & $\int dk_2$.

So choose $(k_1 - k_2)$, etc.

Recurrence formula

Start $\displaystyle\int \exp\left[i\left(k'z + \frac{\bar z}{k'}\right)\right](k'-k)^{-\kappa_0}\,dk' = f_0(z,k)$

$|k'|\to\infty$: $\mathrm{Im}\,(k'z) < 0$ $\qquad |k'| > |k|$

Next $\displaystyle f_1(z,k) = \int f_0(z,k')\exp\left[-i\left(k'a_1 + \frac{\bar a_1}{k'}\right)\right](k'-k)^{-\kappa_1}\,dk'$

$|k'| < |k|$

$|k'|\to\infty$: $\mathrm{Im}\,k'(z-a) < 0$

$$f_{n+1}(z,k) = \int f_n(z,k')\exp\left[-i\left(k'a_{n+1} + \frac{\bar a_{n+1}}{k'}\right)\right](k'-k)^{-\kappa_{n+1}}\,dk'$$

$\mathrm{Im}\,k'(z-\sum a) < 0$

f_n has branch pts at $0,\ a_1,\ a_1+a_2,\ a_1+a_2+a_3, \dots$

The factor $\sim \exp(i\,\bar a_m/k_m)$ does not seem necessary in general. Why should it be there? Only a branch pt $\sim k_m^{\kappa}$ is enough for the last variable $m = N$., and none for the others.

Convergence conditions.

At $z=0$: We lose the factor $\exp[ikz]$

$$\int dk\,(k-l)^{\kappa_{01}}\,k^{\kappa_0} < \infty \quad \rightarrow \quad \kappa_0+\kappa_{01} < -1.$$

At $z=a$:

$$\exp[ikz - ila]\; l^{\kappa_1}\,(k-l)^{\kappa_{01}}\,dl$$

$$\rightarrow \exp[ia(k-l)] \quad .. \quad ..$$

This is first integrated over k, $k>l$. So we are left with $\int l^{\kappa_1}\,dl. \quad \rightarrow \quad \kappa_1 < -1$

Wait: k & l may go to ∞ in different directions, in which case $\underset{min}{k-l} \propto l$ effectively. So the l integration will converge:

$$\int \exp[i(k-l)a]\,(k-l)^{\kappa_{01}}\,dk\;\underset{(l+h)^{\kappa_0}}{\overset{\shortparallel}{k^{\kappa_0}}} \qquad k-l \equiv h$$

or $k = l\xi \qquad k-l = \cancel{l(1-\xi)}\; l(\xi-1)$

$$\int \exp[-il(1-\xi)a]\; l^{\kappa_0+\kappa_{01}}\,(-1+\xi)^{\kappa_{01}}\,\xi^{\kappa_0}\; l\,d\xi \qquad |\xi|>1.$$

This shows convergence ~~for~~ over l integration.

~~What about the case $z=0$~~?

$$\int \cancel{\exp[-il\xi a]} \;\cdots$$

Limit of integer ~~v~~ vorticity:

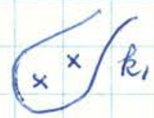
k_0 has ~~po~~ singularities at $\infty, 0$; $k_0 = k_1$ branch pt $\varkappa_{01}$

$k_0 = 0$, $\varkappa_0$

k_1 branch pts $k_0, k_2, 0, \infty$

$\varkappa_{01}\ \varkappa_{12}\ \varkappa_1$

As ~~$\varkappa_{01}$~~ $\varkappa_1, \varkappa_{01} \to$ ~~intege~~ integer ≥ 0, $\int dk_1$ loses singularities to go around:



so $\int \to 0$.

k_0 integration cannot $\to 0$ $\because$ the essential singularity at 0.

Thus the wave fn has the property that if any of the vorticities α_i, $i = 1, \cdots$ becomes integer ≥ 0, it vanishes.

Such a situation will not happen if the exp. factor have terms $\sim \sum \bar{a}_i / l_i$.