Plane wave problem.

First try one vortex with factor $1/(k-k_0)$

$$\Rightarrow \quad i(z+\alpha l) + \frac{\kappa}{k} - \frac{1}{k-k_0} = 0 \qquad |k_0|^2 = m^2$$

$$i(\bar{z}+\alpha k) - \frac{\kappa}{l} = 0$$

$$\Rightarrow \quad l = -i\kappa/(\bar{z}+\alpha k), \text{ also } kl = m^2$$

$$i(\bar{z}+\alpha k) - \kappa k/m^2 = 0 \quad \rightarrow \quad k = i\bar{z}/\left(\frac{\kappa}{m^2} - i\alpha\right)$$

$$\text{or } i\alpha = \left(\frac{\kappa k}{m^2} - i\bar{z}\right)/k$$

Also
$$iz + i\alpha\frac{m^2}{k} + \frac{\kappa}{k} - \frac{1}{k-k_0} = 0$$

$$iz + \left(\frac{\kappa k}{m^2} - i\bar{z}\right)\frac{m^2}{k^2} + \frac{\kappa}{k} - \frac{1}{k-k_0} = 0$$

$$iz - \frac{m^2}{k^2}i\bar{z} + \frac{2\kappa}{k} - \frac{1}{k-k_0} = 0$$

For large $z, \bar{z}$, one sol. $\rightarrow$ $k = k_0$ or more precisely

$$k - k_0 \sim -i/(z-\bar{z}) \quad \text{if } z - \bar{z} \to \infty \text{ (non-forward).}$$

The other sol. ~~$m^2/k^2 \to 1$, $k \neq k_0$ i.e. $k = \pm m$?~~

$$k^2 = m^2\bar{z}/z, \quad k \neq k_0$$

However, the original formula is not symmetric w.r.t. $k \leftrightarrow l$.

$$\int \frac{dk}{k-k_0}\, dl\, \delta(kl-m^2) = \int \frac{dk}{k-k_0}\frac{1}{|k|}$$

$$\text{or } = \int \frac{dl}{|l|}\frac{1}{\frac{m^2}{l}-k_0} = \int \frac{dl}{l}\frac{l}{m^2 - k_0 l} = -\frac{1}{k_0}\int dl/(l-l_0)$$

Take the average $\frac{1}{2}\left(\frac{1}{k-k_0} + \frac{1}{l-l_0}\right)$?

$$\rightarrow \quad \frac{1}{2}\left(\frac{dk}{k-k_0}\frac{1}{|k|} + \frac{dk}{\frac{m^2}{k}-\frac{m^2}{k_0}}\frac{1}{|k|}\right) = \frac{1}{2}\frac{1}{|k|}\left(\frac{1}{k-k_0} - \frac{1}{k-k_0}\frac{kk_0}{m^2}\right)$$

$$= \frac{1}{2}\frac{dk}{|k|}\frac{1}{k-k_0}\left(1 - \frac{kk_0}{m^2}\right)$$

No pole if $k_0 = l_0 = $ real. (forward)

Neither one seems satisfactory.

(c) $\frac{1}{2}\left(\frac{k_0}{k-k_0} - \frac{l_0}{l-l_0}\right) \rightarrow \frac{1}{2}\frac{dk}{|k|}\frac{1}{k-k_0}\left(k_0 + \frac{k k_0 l_0}{m^2}\right) = \frac{1}{2}\frac{dk}{|k|}\frac{1}{k-k_0}(k+k_0)$

This is O.K. $= \frac{dk}{|k|}\left(\frac{k_0}{k-k_0} + \frac{1}{2}\right)$ But $=0$ for $k=-k_0$

so it vanishes in the backward direction!

For the scatt. problem, we have to make sure that no incom. waves exist of form $\exp[-imr]$

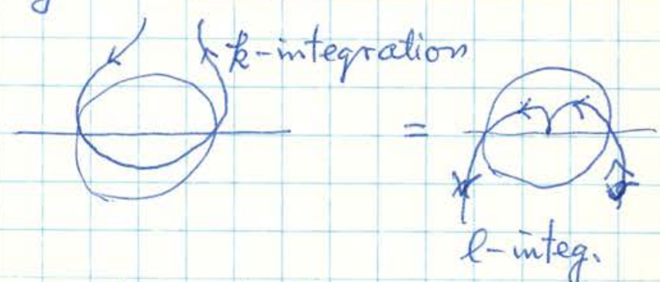
$$kz + \bar{k}\bar{z} = \pm m|z| \quad \text{for} \quad k = \pm m\sqrt{\bar{z}/z}$$

l

This choice can be controlled by a proper k-path.

If the two terms $1/k-k_0$ & $1/l-l_0$ refer to different paths, then there is no sense in combining the two:

$\mathrm{Im}\ kz + l\bar{z} > 0$



The two integrals have different phase factors at the saddle points ± 1 due to the diff. path directions.

Then we have to adjust the relative phases of the Sals so that the contribution at -1 cancel.

Multiple scattering picture:

Scattering by z_1 and then z_2:

$$e^{ik_1(z_2-z_1)}\, e^{ik_2(z-z_2)}$$

On the other hand, our formula reads

$$e^{ik_1(z-z_1)}\, e^{ik_2(z-z_2)} = e^{i(k_1+k_2)z - ik_1z_1 - ik_2z_2}$$

Let $k_1+k_2 = k$

$$k_1z_1 + k_2z_2 = k_1(z_1-z_2) + (k_1+k_2)z_2 = k_1(z_1-z_2) + kz_2$$

So $\exp = \exp i[k(z-z_2) + ik_1(z_2-z_1)]$

Correspondence is O.K.

In the usual propagator picture, we take $1/kl-m^2$

and integrate over k & l. Do we need it here?

If so, the effective propagator is

$$(k/l)^{\kappa}/(kl-m^2)$$

Q. If we expand $1/(kl-m^2)$, we get terms

$$\sim k^{n+\kappa}\, l^{n-\kappa}$$

Define $\tilde{G}(k,l) = (k/l)^{\kappa}/(kl - m^2)$

Then $(kl - m^2)\,\tilde{G}(k,l) = (k/l)^{\kappa}$

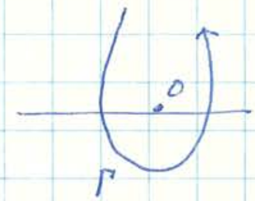
"Fourier" repr:

$$\int (k/l)^{\kappa} e^{i(kz + l\bar{z})}\, dk\, dl = \int k^{\kappa} e^{ikz} \times \int l^{-\kappa} e^{il\bar{z}}\, dl$$

Integration path $\mathrm{Im}\, kz > 0$.

Now consider

$$f_{\kappa}(z) = \int e^{ikz} k^{\kappa}\, dk = z^{-\kappa-1} \int_{\mathrm{Im}\, u > 0} e^{iu} u^{\kappa}\, du$$



The $\int$al must be a Γ fn.

$$I_{\kappa} \equiv \int e^{iu} u^{\kappa}\, du = \frac{u^{\kappa+1}}{\kappa+1} e^{iu}\Big| - \frac{i}{\kappa+1}\int e^{iu} u^{\kappa+1}$$

$$= \frac{-i}{\kappa+1} I_{\kappa+1}$$

~~Also $I_{\kappa+1}$ ...~~

$\Rightarrow \quad I_{\kappa+1} = i(\kappa+1) I_{\kappa} \qquad \Gamma(\kappa+1) = \Gamma(\kappa)\,\kappa$

$\Rightarrow \quad \cancel{1/I_{\kappa}} \quad I_{\kappa} = C\,\Gamma(\kappa+1)\, i^{\kappa}$

Or else $\quad I_{\kappa} = C\, i^{-\kappa}/\Gamma(-\kappa)$

The second one is the correct form since for κ = integer ≥ 0,

$I_{\kappa} = 0$

In any case

$$(kl - m^2)\,\tilde{G}(k,l) = (k/l)^{\kappa}$$

or $-(\nabla_2^2 + m^2)\,G(z,\bar z) = f_{\kappa}(z)\, f_{-\kappa}(\bar z) \propto \cancel{(\bar z/z)^{\kappa+1}}\; z^{-\kappa-1}\bar z^{-\kappa-1}$

Then $$(z/\bar z)^{\kappa+1}(\nabla_2^2 + m^2)\,G(z,\bar z) = \text{const}$$

$$\nabla^2 = 2\frac{\partial}{\partial z}\frac{\partial}{\partial \bar z}$$

so $$\left[\cancel{2}\left(\frac{\partial}{\partial z} - \frac{\kappa+1}{z}\right)\left(\frac{\partial}{\partial \bar z} + \frac{\kappa+1}{z}\right) + m^2\right] G'(z,\bar z) = \text{const}$$

$$G' = (z/\bar z)^{\kappa+1}\, G$$

$$\text{Const} = c/\Gamma(\kappa)\,\Gamma(-\kappa) = c/\Gamma(\kappa) \times \frac{-\sin\pi\kappa}{\pi}\,\Gamma(1+\kappa)$$
$$= -\kappa\frac{c}{\pi}\sin\pi\kappa$$

In any case, const $= 0$ for κ integer

Divide $\tilde G$ by an approp. fn of κ so that for $\kappa = 0$, we have $\tilde G = 1/(kl - m^2)$, the usual propagator

According to our complex integ. formula,

$$\int 1\, e^{i(kz + l\bar z)}\, dk\, dl = 0$$

instead of the $\delta(z)\,\delta(\bar z)$ that one might expect.

This could be resolved by a suitable def. of the integral when $z = \bar z = 0$.

But what is the meaning of const on the r.h.s. ?

Rewrite $$G(z,\bar z) = \int e^{i(kz+l\bar z)} \frac{\cancel{k_{y}}}{\cancel{(\bar z/z)}} (k/l)^{\kappa} / (kl - m^2)\, dk\, dl$$

$$= \int e^{i(u+v)} (u/v)^{\kappa} \Big/ \left(\frac{uv}{z\bar z} - m^2\right) du\, dv\, \cancel{(\bar z/z)}^{\kappa+1}$$

$$\times\ z^{-\kappa-1}\, \bar z^{+\kappa-1}$$

Probably I had made an error: should have written

$$(k/l)^{\kappa} \frac{dk}{k}\frac{dl}{l} \quad \text{or} \quad = k^{\kappa-1} l^{-\kappa-1}$$

~~or rewrite in the original formula~~ $\cancel{k}$

Thus $$\cancel{f}\,(kl-m^2)\,\tilde G(kl) = k^{\kappa-1} l^{-\kappa-1}$$

$$-\left(\frac{\nabla^2}{2} + m^2\right) G(z,\bar z) = f_{\kappa-1}\, f_{-\kappa-1} \propto z^{-\kappa} \bar z^{-\kappa} = (\bar z/z)^{\kappa}$$

and $$\left[\left(\frac{\partial}{\partial z} - \frac{\kappa}{z}\right)\left(\frac{\partial}{\partial \bar z} + \frac{\kappa}{\bar z}\right) + m^2\right] G'(z,\bar z) = \text{const}$$

$$G' = (\bar z/z)^{\kappa} G.$$

$$G(z,\bar z) = \int e^{i(kz+l\bar z)} (k/l)^{\kappa} / (kl-m^2) \frac{dk}{k}\frac{dl}{l}$$

$$= \int e^{i(u+v)} (u/v)^{\kappa} \Big/ \left(\frac{uv}{z\bar z} - m^2\right) \frac{du}{u}\frac{dv}{v} \times (\bar z/z)^{\kappa}$$

In this form, the u & v integration paths are fixed. $\operatorname{Im} u$, $\operatorname{Im} v > 0$

This is a funny function: the source is a constant.

The usual Green's fn. For free field,

$$G(\vec{r},\vec{r}') = \sum_k \langle r|\vec{k}\rangle\langle\vec{k}|r'\rangle \Big/ (\vec{k}^2 \mp m^2)$$

$\sum_k$ over all $\vec{k}$ $\qquad (\nabla^2 \mp m^2)\langle r|k\rangle = -(k^2 \mp m^2)\langle r|k\rangle$

Actually $\quad \langle r|k\rangle = e^{i\vec{k}\cdot\vec{r}}$

$$\int e^{ik\cdot(r-r')} \Big/ (k^2 \mp m^2)\, d\overset{2}{k}$$

In our case, use the scat. ampl. formula

$$\langle r|k_0\rangle = \int e^{i(kz + \frac{m^2}{k}\bar{z})}\, k^{\kappa} \big/ (k-k_0) \cdot \frac{dk}{k}$$

multiply $\quad \langle r|k_0\rangle \times \langle k_0|r'\rangle$ and $\int dk_0/k_0$

where $\bar{k}_0 k_0 = m^2$ so $\bar{k}_0 = m^2/k_0$

$$\oint \frac{1}{k-k_0}\,\frac{1}{l - \frac{m^2}{k_0}}\, dk_0/k_0 = \oint \frac{1}{k-k_0}\,\frac{1}{k_0 l - m^2}\, dk_0$$

$$= \oint\left(\frac{1}{k-k_0}\,\frac{1}{kl-m^2} + \frac{1}{k_0 l - m^2}\,\frac{1}{k - \frac{m^2}{l}}\right)$$

$$= 2\pi i \times \begin{cases} \dfrac{-1}{kl-m^2} & \text{if } |k| < m,\ |l| > m \\[2ex] \dfrac{1}{kl-m^2} & \text{if } |k| > m,\ |l| < m \\[2ex] 0 & |k| > m,\ |l| > m \text{ or } |k| < m,\ |l| < m \end{cases}$$

~~As we~~ Next we integrate over $|k_0| = m$ as we did integrate over $\vec{k}$ in the free field case. We then get a factor

$$2\pi i \cdot \cancel{\frac{(|k|-|l|)}{kl-m^2}}\,[\theta(|k|-m)\,\theta(m-|l|) - \theta(m-|k|)\,\theta(|l|-m)]$$

$$\times \int_{|k|}^{|l|} \frac{dm^2}{kl-m^2} = \cancel{\frac{1}{\sqrt{kl}} \ln \frac{\sqrt{kl}+m}{\sqrt{kl}-m}\Big|_{|k|}^{|l|}} \quad \ln\frac{kl-|k|^2}{kl-|l|^2}$$

Path integral form.

$$\int L = \sum' \frac{(x_n - x_{n-1})^2}{\Delta} \quad \text{or} \quad \int \dot{x}^2 \, dt$$

Classical path $\quad \ddot{x} = 0 \quad$ or $\quad x = vt + c$

$$x_n - x_{n-1} = v\Delta \qquad \sum \frac{(\Delta x)^2}{\Delta} = \sum' v^2 \Delta = T v^2$$

$$v = (x_N - x_1)/T \qquad\qquad = (x_N - x_1)^2/T$$

Fluctuations $\quad x = x_{cl} + y \quad \int \dot{x}^2 = \int \dot{x}_{cl}^2 + \int \dot{y}^2$

$$\int \exp[i \int \dot{y}^2 dt] \, \mathcal{D}y = \int \exp[i \sum' \frac{(y_n - y_{n-1})^2}{\Delta}] (\mathcal{D} y_n)$$

$$y_1 = y_N = 0$$

~~$y_1 = 0$ y_N~~ This factor corresponds to transition $x = 0 \to x = 0$.

This should give a factor $1/\sqrt{T}$.

Next the broken path:

Again $\vec{x}_i = \vec{v}_i t + c \qquad i = 1, 2$

$$\int L = \frac{R_1^2}{T_1} + \frac{R_2^2}{T_2} = v_1 T_1 + v_2 T_2$$

Minimize w.r.t. T_i, $\quad T_1 + T_2 = T$

$$\rightarrow \frac{R_1^2}{T_1^2} = \frac{R_2^2}{T_2^2} \quad \text{or} \quad v_1 = v_2 = \frac{R_1 + R_2}{T}$$

$$\int L = (R_1 + R_2)^2 / T$$

$$T_i = R_i / R \, T$$

Fluctuations: $\quad \dfrac{1}{\sqrt{T_1}} \dfrac{1}{\sqrt{T_2}} = \dfrac{1}{T} \cdot \dfrac{R}{\sqrt{R_1 R_2}}$

These must be for those fluctuations which pass through pt P.

Quantum Brownian motion.

Take an A-B medium at a given temperature. We consider A-B "particles" to be free noninteracting gas. in ~~a~~ a grand canonical ensemble. Q: How will an electron behave?

Classical eq: $\quad m\ddot{x}_i = eF_{ij}\dot{x}_j\,, \qquad F_{12}(x) = \sum_n \kappa_n \delta^2(x-x_n)$

and $\quad \langle \kappa_n \rangle$ = ensemble average $= 0$.

but $\quad \langle \kappa_n^2 \rangle \neq 0$

The Lagrangian for the interaction is

$$\langle \exp[ie\int A_\mu dx^\mu] \rangle = \langle \exp[ie\int A_i\, dx^i] \rangle$$

$$A_i(x) = \sum_n A_i^{(n)}(x) \qquad n \text{ for vortex } \# n.$$

So the question is

$$\langle \exp[i\int A_i dx_i] \rangle \quad \text{for each vortex for a given path.}$$

Let's first solve $m\ddot{x} = 0$ for a classical path which is straight. Then consider quantum fluctuations. So we take a straight path.

Then $\quad U = \exp[i\int A\cdot dx] = U_0 \cdot \exp[i\sum F_{12}\,\Delta\sigma_{12}]$

$$\rightarrow \langle U \rangle = U_0 \langle \exp[i\sum F_{12}\,\Delta\sigma_{12}] \rangle$$

The keyboard diff eq.:

$$\left\langle \frac{\delta^2 U}{\delta\sigma^2} \right\rangle = \langle -(eF_{12})^2 U \rangle \qquad \text{if } \langle F_{12} U \rangle = \text{const} \langle U \rangle$$

$$\approx -\langle (eF_{12})^2 \rangle \langle U \rangle \quad ?$$

A sol. $U \approx U_0 \exp [\pm i \sum \underbrace{|eF_{12}|}_{\text{const}} \Delta\sigma_{12}]$

$\Delta\sigma_{12} = hw$

At each pt on the straight line, there is a freedom of perpendicular displacements h ~~for~~ with width w.

$w = (w/\Delta)\, \Delta t = v \Delta t$ where $v =$ ~~t~~ R/T.

Is the above U correct? One may also say that

$$\langle \exp [i F_{12} \Delta\sigma_{12}] \rangle = \sum_k \exp [iK]\, P(k)\, \Delta\sigma_{12} \quad ?$$

$P(k)(\Delta\sigma_{12})$: probability of having a flux k inside the area $\Delta\sigma_{12}$

This does not sound right.

∃ a basic problem: Make $\Delta\sigma_{12} \to$ small enough ~~so the~~ and $\langle F \rangle$ finite.

then $\exp \to 1 + i F_{12} \Delta\sigma_{12}$ and $\langle F_{12} \rangle = 1$ for symmetry.

If ∃ neighbor correlations, one could get

$$\langle \Pi (1 + i F_{12} \sigma_{12}) \rangle \to 1 - \sum \langle F_{12} F_{12}' \rangle \sigma_{12} \sigma_{12}' / 2$$

$$\tilde{\to} \exp [-\underbrace{\sum \langle e^2 F^2 \rangle}_{\times \text{ const} > 0} / 2\, \Delta\sigma_{12}] \quad \text{if the correlation is short range.}$$

But this has no i compared to the above $U \approx \exp [\pm i \sum |eF_{12}| \Delta\sigma_{12}]$

Wrong: $\sum \langle F_{12}' F_{12}' \rangle \, \Delta\sigma_{12} \, \Delta\sigma_{12}' \rightarrow$ Const. $h^2 \Delta w$

The effective Lagrangian for the string then is imaginary!

On the other hand, one may take

$U \sim U_0 \exp[\, i \sum' | e F_{12} \Delta\sigma_{12} | \,]$ with a definite ~~goo~~ sign for all points.

corresponding to a potential: $\sum_n |h_n| w_n = \sum_n |h_n| v \, \delta t_n$

The fluctuations $h(t)$ has an effective total L:

$$\cancel{m\dot{h}^2} \quad \frac{m}{2} \dot{h}^2 + c|h|$$

$\langle \exp[\, i F_{12} \Delta\sigma_{12} \,] \rangle = \sum \{ \cos(|F_{12}| \Delta\sigma_{12}) \, P(|F_{12}|)^{\Delta\sigma_{12}}$ if P is

symmetric. $\rightarrow$ loss of additivity or continuum limit:

$$\langle \exp[\, i \sum_n F_n \Delta\sigma_{12} \,] \rangle \rightarrow \exp[\, \sum_n \ln \cos(\bar{F}_{12} \Delta\sigma_{12_n}) \,]$$

$$\rightarrow \exp[\, -\frac{1}{2} \sum_n' \bar{F}_{12_n}^2 (\Delta\sigma_{12})_n^2 \,]$$

Stat. Mech. calculation

Probability for a vortex to be within a volume $v \propto v$.
It contributes a factor $\exp(i\alpha)$.
N particle system

$$\sum_n \frac{v^n (V-v)^{N-n}}{n!\,(N-n)!} e^{i\alpha n} = [(V-v) + e^{i\alpha} v]^N / N!$$
$$= V^N [1 + \frac{v}{V}(e^{i\alpha}-1)]^N / N!$$

$$\sum_N [1+\frac{v}{V}(e^{i\alpha}-1)]^N / N! = \exp[1+\frac{v}{V}(e^{i\alpha}-1)]$$

For negative vortices $\qquad \exp[1+\frac{v}{V}(e^{-i\alpha}-1)]$

Product $\qquad C \exp[\frac{2v}{V}(\cos\alpha - 1)]$

This has the extensive property.

In our case, we replace $\frac{v}{V} \to |hw|$

The above form is different from the naive expectation

$\exp[i|F|\,hw]$.

Reason: the probability is peaked around $n=0$.
since v/V is small. Then the factorization
$\langle F^2 U \rangle \sim \langle F^2 \rangle \langle U \rangle$ does not apply.

If chemical pot. μ is included,

$$\sum \to \sum \mu^N [1+\frac{v}{V}(e^{i\alpha}-1)]^N/N!$$

$$\to \exp[\mu \frac{v}{V}(e^{i\alpha}-1)]$$

$\mu/V \to \rho$, the vortex density.

The effective action

$$\exp\left[\, i \sum \dot{x}^2 \Delta\cancel{t} + i \sum A \cdot dx \,\right]$$

$$\rightarrow \int \frac{1}{\sqrt{T}} \exp\left[\, i\left(\frac{R^2}{T} + \sum \dot{h}_i^2 \Delta\cancel{t}\right) - \sum |h_i \omega| \cancel{\beta}\lambda \,\right], \quad \lambda = 2\rho\,\cancel{\cos}(1-\cos\alpha)$$

$$\times (\mathcal{D} h)$$

~~The~~ One degree of fluctuation has been integrated over.

$$\sum |h_i| \omega = \sum |h_i| v \Delta \qquad v = R/T \text{ fixed}$$

"Schrödinger eq" to be solved for the h-degree of freedom

$$i \dot{\psi} = \cancel{\beta}\left(p_h^2 - i v |h| \lambda\right) \psi$$

If Euclidean space was used from the beginning,

$$\dot{\psi} = \left(\nabla_h^2 - \lambda |h| v\right) \psi$$

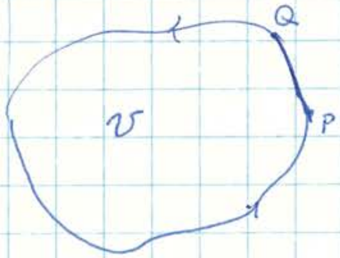
and ~~take~~ one takes the value $\psi(h,t)\big|_{t=T,\, h=0}$

This corresponds to 1-dim. motion under a linear potential.

7/2

Paradox:

Computing the contributions from vortices, we ~~come~~ took a straight line as the reference pt. But we have to imagine it as a border line of a large area v



$$Z = \exp[\mu(1-\tfrac{v}{V}) + \mu \tfrac{v}{V} e^{i\alpha}]$$

For the straight line, let $v = v_0$.

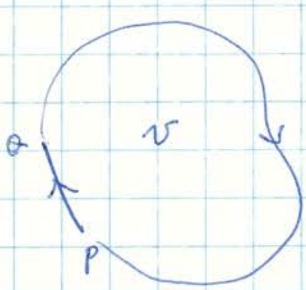
$$Z/Z_0 = \exp[\mu(\tfrac{v_0 - v}{V}) + \mu \tfrac{v-v_0}{V} e^{i\alpha}]$$

$$= \exp[\mu \tfrac{v-v_0}{V}(e^{i\alpha}-1)]$$

$$\sum_{\pm\alpha} = \exp[2\mu \tfrac{v-v_0}{V}(\cos\alpha - 1)]$$

It depends on the sign of $v - v_0$!

Clearly we are doing something strange. It is like the derivation of the H-theorem. The answer depends on the starting v.



$$Z' = \exp[\mu(1-\tfrac{v}{V}) + \tfrac{v}{V} e^{-i\alpha}]$$

$$Z'/Z_0' = \exp[\mu \tfrac{v-v_0}{V} e^{-i\alpha}]$$

Here $v - v_0 \to -(v - v_0)$ compared to the earlier def.

$$Z_0 = \exp[\mu(1-\tfrac{v}{V}) + \mu\tfrac{v}{V} e^{i\alpha}] = \exp[\mu(\tfrac{v'}{V} + \tfrac{v}{V} e^{i\alpha})$$

$$v' = V - v$$

$$Z_0' = \exp[\mu(\tfrac{v}{V} + \tfrac{v'}{V} e^{-i\alpha})]$$ if we take the complementary space

They are simply related by a "gauge" $\mu \to \mu e^{\pm i\alpha}$ ~~$\exp[\mu e^{\pm i\alpha}]$~~

Product of $\pm\alpha$: $Z = \exp[2\mu \frac{v'}{V} + 2\mu \frac{v}{V}\cos\alpha]$

$$Z' = \exp[2\mu \frac{v}{V} + 2\mu \frac{v'}{V}\cos\alpha]$$

$$Z_0 = \exp[2\mu \frac{v_0'}{V} + 2\mu \frac{v_0}{V}\cos\alpha]$$

$$Z_0' = \exp[2\mu \frac{v_0}{V} + 2\mu \frac{v_0'}{V}\cos\alpha]$$

They are not related by a gauge.

We should have used ~~a~~ different μ's for $\pm\alpha$.

At any rate, Z & Z' are related by $\exp[\pm 2\mu\cos\alpha]$ now.

The origin of ambiguity. In the A-B gauge, the potential is multi-valued. Essentially $\int A\cdot dx$ is the ~~angle~~ angular distance PQ as viewed from the vortex pt. To make this $\to 0$ for distant objects, we have to take two different branches on different sides of line PQ.

A similar situation arose in the case of the monopole gas of Polyakov.

Q: Can the H-Theorem be derived in a similar way? There must be a potential between two events.

7/2. Objection by Nino.

$$\int A\cdot dx \to \quad A(vt+y)\cdot(v+\dot{y})\,dt$$

$$= \int A(vt)\cdot v\,dt + \int A_i(vt)\cdot \dot{y}_i\,dt + \int \frac{\partial A_k}{\partial x_i} y_i v_k\,dt$$

$$+ O(y^2)$$

$$= \int A\cdot v\,dt \underbrace{- \int \dot{A}_i y_i\,dt + \int \frac{\partial A_k}{\partial x_i} y_i v_k\,dt}_{F_{ik}\, y_i v_k\, dt}$$

$O(y^2)$ terms

$$\int \frac{1}{2}\partial_i\partial_j A_k\, y_i y_j \cancel{\dot{y}_k}\, v_k dt + \int \partial_i A_k\, y_i \dot{y}_k\,dt$$

$$\downarrow$$

$$- \partial_i \dot{A}_k\, y_i y_k - \partial_i A_k\, \dot{y}_i y_k$$

$$\downarrow \qquad\qquad \downarrow$$

$$- \partial_i\partial_j A_k v_j\, y_i y_k \quad -(F_{ik} + \partial_k A_i)\,\dot{y}_i y_k$$

$$\text{2nd term} = \frac{1}{2}\int\left(-\partial_i\partial_j A_k v_j\, y_i y_k - F_{ik}\,\dot{y}_i y_k\right)$$

$$\to \quad \int = \int \frac{1}{2}\partial_i F_{jk}\, y_i y_j v_k\,dt - \frac{1}{2}\int F_{ik}\,\dot{y}_i y_k\,dt$$

But $\quad \partial_i F_{jk} = \partial_j F_{ik} + \partial_k F_{ji}$

$\times y_i y_j \quad \to \quad = \partial_j F_{ik}$

such an expansion cannot be made for singular A.

F is a sum of δ-fns.