Jan. 79.

I re-examine the Coulomb eq. in non Abelian cases

$$H = \frac{1}{2}(E^2 + B^2)$$

$$\mathrm{Div}\, E = \partial_i E_i + A_i \times E_i = \rho \qquad \rho : \text{external.}$$

$A_i \times E_i$ is the isospin operator $-\vec{I}$

Any vector $\vec{E}_\ell = -\vec{\nabla}\varphi_0 + \vec{\nabla}\times \vec{a} = E_\ell + E_t$

$$\rightarrow \quad -\nabla^2 \varphi_0 = I + \rho$$

$$\rightarrow \quad \int H = \frac{1}{2}\int (E_\ell^2 + E_t^2 + B^2)$$

$$\rightarrow \sim \frac{1}{2}\int [(\nabla\varphi_0)^2 + E_t^2 + B^2]$$

$$= \frac{1}{2}\int (I+\rho)\frac{1}{-\nabla^2}(I+\rho) + \cdots$$

$$E_t^2 = E^2 - \frac{(\nabla\cdot E)^2}{\nabla^2} = (\nabla\times a)^2$$

How is this related to A_μ?

$$E = \cancel{-D_i A_0 - D_0 A_i = -\partial_i A_0 - A_i \times A}$$
$$-\partial_i A_0 - \partial_0 A_i + A_0 \times A_i$$

If $\partial_i A_i = 0 \longrightarrow \nabla\cdot E = -\nabla^2 A_0 + \partial_i A_0 \times A_i$ should be $= -\nabla\varphi_0$

$$\nabla\times(\nabla\times E) = -\partial_0 \nabla\times(\nabla\times A) + \nabla\times(\nabla\times)(A_0\times A_i)$$

$$= + \nabla^2 \dot{A} + \nabla_i \nabla\cdot(A_0\times A_i) - \nabla^2(A_0\times A_i)$$

$$= \nabla^2 D_0 A_i + \nabla_i \nabla\cdot(A_0\times \vec{A})$$

Any way we have to solve for A_0 in terms of φ_0

Questions:

I separated E_ℓ and expressed it in terms of $\rho + I$.

Can I then simply define $E_t = E - E_\ell$, so the indep.

variables are A_i, Π_i ?

$$= E - \frac{\nabla(\nabla\cdot E)}{\nabla^2}$$

$$\Pi_i = E_i$$

If so, no unusual properties seem present in H.

But is this correct?

From $H = \frac{1}{2}(\Pi^2 + B^2)$ $B = \nabla\times A + A\times A$

$\rightarrow$ $\dot{A}_i = \Pi_i$ $\Rightarrow$ Not correct.

$\dot{\Pi}_i = -(D\times B)_i$

Check: $\delta\mathcal{L}/\delta\dot{A}_i = -E_i = \dot{A}_i + \nabla_i A_0 - A_0\times A_i$

so $\dot{A}_i = \Pi_i - \nabla_i A_0 + A_0\times A_i = \Pi_i - D_i A_0$

$\Pi\cdot\dot{A} = \Pi_i^2 - \Pi_i D_i A_0 \rightarrow \Pi_i^2 + D_i\Pi_i A_0$

$\widetilde{=}\ \Pi_i^2 - \rho A_0$ weak equality.

So we cannot ignore $D\Pi A_0$.

If A_0 is regarded as a Lagrange multiplier, $\rightarrow D\Pi + \rho = 0$

If $\Pi \rightarrow \Pi_\ell + \Pi_t$

Check of consistency of $D\cdot\Pi - \rho = 0$.

$$H = \tfrac{1}{2}(\Pi^2 + B^2) + (D\cdot\Pi - \rho)A_0$$

$$D\cdot\Pi = \partial_i\Pi_i + A_i\times\Pi_i = \partial_i\Pi_i + I$$

I commutes with $\Pi^2 + B^2$

$$[\Pi_i, \int(\tfrac{1}{2}B^2)] = -i\frac{\delta}{\delta A_i}\int\tfrac{1}{2}B^2 = -\vec{D}\times\vec{B} \quad \text{or} \quad D_j B_{ji}$$

$$\vec{D}\cdot(\vec{D}\times\vec{B}) = (D\times D)\cdot B = \vec{B}_i\times B_i = 0.$$

$$D_i D_j B_{ji} = \tfrac{1}{2}[D_i, D_j]B_{ji} = -\tfrac{i}{2}[B_{ij}, B_{ji}] = 0. \quad \text{Yes}$$

But $\partial_i(D_j B_{ji}) = \partial_i(A_j\times B_{ji}) \qquad B_{ji} = \partial_j A_i - \partial_i A_j + A_i\times A_j$

$$\to (\partial_i A_j)\times(A_j\times A_{ji}) + A_j\times\partial_i B_{ji}$$

Why should it be zero?

Probably I was wrong to claim $[I, B^2] = 0$ since B is not strictly local w. r. to A_i?

Since $\int I\, d^3x$ does commute with H, $[I, B^2]$ must be a pure divergence, which in fact is right because $\sim \partial_i\Pi_i$.

Jan 10.

I should study the ~~struct~~ properties of phase factor U, in the hope of dealing with the infinities.

$$U = \exp[i\int A\cdot dx]$$

Interpretation : 1) a heavy ~~mass~~ particle moving along a path.

2) creation of a flux line.

$\langle U[\sigma]\rangle$ This violates gauge inv. for an ordinary vac.?

The generator of gauge transf. is ~~Div E or~~ Div Π

So if $\mathrm{Div}\,\Pi\,|0\rangle = 0$, it would be O.K.

But in fact we usually replace it with $:\mathrm{Div}\,\Pi:$

Evaluate $\langle U\rangle$ any way :

$$= \exp[-\tfrac{1}{2}\iint\langle A_\mu(x)\,A_\nu(y)\rangle\, dx_\mu\, dx_\nu] \quad \text{~~if~~ Abelian.}$$

$$= \exp[-\tfrac{1}{2}\iint \Gamma_{\mu\nu} D(x-y)\, dx_\mu\, dx_\nu]$$

Gauge dependent term $\propto$ $\Gamma_{\mu\nu} \approx \dfrac{\partial_\mu\partial_\nu}{\Box}$

$$\oint\!\!\oint \partial_\mu\partial_\nu \tfrac{1}{\Box} D(x-y)\, dx\, dy = 0 \quad \text{formally.}$$

The main term involves

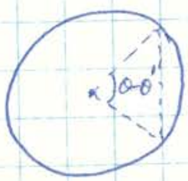
$$\oint\oint \frac{1}{(x-y)^2}\, dx\cdot dy$$

4-dim version of the self-energy of a current loop.

$$\oint\oint e^{ik\cdot(x-y)}\, dx\cdot dy$$

Evaluation for a circle

$$dx_1 = -r\sin\theta\, d\theta$$
$$dy_2 = r\cos\theta\, d\theta$$



$$dx\cdot dy = r^2(\cos\theta\cos\theta' + \sin\theta\sin\theta')\, d\theta\, d\theta'$$
$$= r^2\cos(\theta-\theta')\, d\theta\, d\theta'$$

$$(x-y)^2 = 4r^2\sin^2\left(\frac{\theta-\theta'}{2}\right)$$

$$\iint \frac{dx\cdot dy}{(x-y)^2} = 2\pi\int_0^{2\pi}\frac{\cos\theta}{4\sin^2\frac{\theta}{2}}\, d\theta = 2\pi\int_0^{2\pi}\frac{1-2\sin^2\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}}\, d\theta$$

$$= \frac{2\pi}{4}\int_0^{2\pi}\frac{1}{\sin^2\frac{\theta}{2}}\, d\theta - 2\pi^2$$

This blows up linearly.

Masslike cutoff:

$$\int\frac{1}{(x-y)^2+\kappa^2}\, dx\, dy = 2\pi\int\frac{\cos\theta\, d\theta}{4\sin^2\frac{\theta}{2}+\frac{\kappa^2}{r^2}}$$

$$= 2\pi\int_0^{2\pi}\frac{\cos\theta\, d\theta}{2(1-\cos\theta)+\frac{\kappa^2}{r^2}} \sim O\left(\frac{r}{\kappa}\right)$$

Actually κ is the distance between two parallel rings

Case of semi-infinite line in $z = [0,\infty)$

$$\int_0^\infty\int_0^\infty \frac{dz\,dz'}{(z-z')^2} = -\int_0^\infty \frac{dz}{z} = -\ln z\Big|_0^\infty \ ?$$

$$\int_0^\infty\int_0^\infty \frac{dz\,dz'}{(z-z')^2+\kappa^2} \qquad z-z' = t, \quad \frac{z+z'}{2} = s$$

$$dz\,dz' = dt\,ds$$

$$= \int_0^\infty ds \int_{-s}^{s} dt\, \frac{1}{t^2+\kappa^2} \qquad -s \le t \le s$$

$$= \int_0^\infty ds\, \frac{2}{\kappa}\tan^{-1}\frac{s}{\kappa}$$

linear div. from large s
log div. small s

Subtraction : $\int_0^\infty ds\, \frac{2}{\kappa}\left(\tan^{-1}\frac{s}{\kappa} - \frac{\pi}{2}\right)$ log div only small s

$\int_0^\infty ds\, \frac{2}{\kappa}\left(\tan^{-1}\frac{s}{\kappa} - \frac{\pi}{2} - \frac{\kappa}{s}\right)$ log div for large s.

$\cancel{\frac{d}{d\kappa}}$:

The linear divergence is reasonable : $\propto$ length.

The phase factor may be regarded as an action integral for a scalar field. So let us consider

$$\mathcal{L} = \frac{m}{2}\dot{x}^2 + g\,\dot{x}\cdot A$$

If integrated over path, this will give a propagator $\overset{(A)}{\Delta}(x,y)$ in the presence of field A.

So we replace U with Δ:

$$M(x,y) \equiv \bar{q}(x)\,\overset{(A)}{\Delta}(x,y)\,q(y)$$

$\Delta(x,y)$ is exponential $\sim \exp(-mr)$ for spacelike separation

Actually we think of U as a 3-dim. spacelike world line.

Δ should be appropriate to a 3-dim field theory ?

$$M(x,y) = \bar{q}(x)\langle\varphi(x)\,\varphi^\dagger(y)\rangle q(y)$$

$\langle \exp[i\int A\cdot dx]\rangle \sim \exp[-\iint \Delta \, dx\,dy]$

and Δ is essentially real : This is ~~the~~ a possible way of making $B^2 < 0$. But anyhow the field A is created by the string itself. So we must ~~say~~ solve for eqs of the type

$D_\mu F_{\mu\nu} = j_\nu \qquad j_\nu$: string.

Q: Can one integrate instanton sol. over a continuous string of poles ?

$$D_\mu u = \partial_\mu u + i u A_\mu$$

$$J_\mu = \sum_n u_n^+ \overleftrightarrow{D}_\mu u_n = 0 \quad \to i\left[\sum u_n^+ u_n, A_\mu\right] + \sum u_n^+ \overleftrightarrow{\partial}_\mu u_n = 0$$

$$A_\mu = \frac{i}{2\sum u_n^+ u_n} \sum_n u_n^+ \overleftrightarrow{\partial}_\mu u_n$$

$$u_n = \lambda \frac{x_\mu \sigma_\mu}{x^2} = \lambda \frac{x_4 + i\vec{\sigma}\cdot\vec{x}}{x^2} \qquad u^+ u = \frac{\lambda^2}{x^2} = f$$

$$x_n \to x - z_n$$

$$\int \frac{\lambda(z)^2}{(x-z)^2}\, dz : \qquad z \text{ along the } z\text{-axis}$$

$$\lambda = \text{cont}: \int_{-\infty}^{\infty} \frac{dz}{a^2+z^2} = \frac{\pi}{a}$$

$\frac{1}{2} u_n^+ \overleftrightarrow{\partial_\mu} u_n$: $\quad \partial_\mu u_n = \sigma_\mu / x^2 - 2\sigma\cdot x\, x^\mu/(x^2)^2$

$$u^+ \partial_\mu u = \cancel{\frac{x}{x^2}} \frac{\tilde{\sigma}\cdot x\, \sigma_\mu}{(x^2)^2} - \frac{2x^\mu}{(x^2)^2}$$

$$\partial_\mu u^+ u = \frac{\tilde{\sigma}_\mu \sigma\cdot x}{(x^2)^2} - \frac{2x^\mu}{(x^2)^2}$$

$$\frac{1}{2} u^+ \overleftrightarrow{\partial_\mu} u = \frac{1}{2} \frac{\tilde{\sigma}\cdot x\, \sigma_\mu - \tilde{\sigma}_\mu \sigma\cdot x}{(x^2)^2}$$

$$\int \frac{\tilde{\sigma}\cdot(x-z)\sigma_\mu}{[(x-z)^2]^2} dz =: \cancel{\frac{1}{2}\int \frac{\partial}{\partial z_\mu} \frac{\sigma}{(x-z)^2} \cdot \tilde{\sigma}_\mu \sigma_\mu\, dz} :$$

$$\int_{-\infty}^{\infty} \frac{\tilde{\sigma}\cdot x_\perp}{(x_\perp^2 + (x-z)_{\parallel}^2)^2} dz = \cancel{\frac{\pi}{2} \frac{\tilde{\sigma}\cdot x_\perp}{|x_\perp|}} - \tilde{\sigma}\cdot x_\perp \frac{\partial}{\partial(x_\perp^2)} \frac{\pi}{|x_\perp|} = \frac{\pi}{2} \tilde{\sigma}\cdot x_\perp \frac{1}{|x_\perp|^3}$$

$$\int \frac{\tilde{\sigma}\cdot(x-z)_{\parallel}}{''} dz = 0$$

So we have $\frac{1}{2}\int dz\; u^+ \overleftrightarrow{\partial_\mu} u = \frac{\pi}{4}\left(\frac{\tilde{\sigma}\cdot x_\perp}{|x_\perp|^3} \sigma_\mu - \tilde{\sigma}_\mu \frac{\sigma\cdot x_\perp}{|x_\perp|^3} \right)$

Also compute $F_{\mu\nu} \sim f(1-f) \partial_\mu \hat{u}^+ \partial_\nu \hat{u} + \partial_\mu f\, \underbrace{\hat{u}^+ \partial_\nu u}$

A_μ is ~~the old A~~ similar to the old A_μ evaluated ~~at~~ in the plane $\perp$ to the axis.

$$A_{\mu\perp} \propto: \frac{\pi}{2} \hat{u}^+ \partial_i \hat{u} \frac{1}{|x_\perp|} \qquad \hat{u} = \sigma\cdot x_\perp / |x_\perp|$$

$$A_{\mu\parallel} \propto \frac{\pi}{2} \hat{u}^+ \sigma_{\mu\parallel} \frac{1}{x_\perp^2}$$

I can choose $\parallel$ to be x_0 - direction $\rightarrow \sigma_0 = 1$

$$u = i\sigma_i \hat{x}_i$$

$\cancel{\partial u^{+} \partial u}$

$$A_\mu = \begin{cases} \dfrac{i\lambda\pi/2}{1+\frac{\lambda\pi}{r}}\, \hat{u}^{+}\partial_i \hat{u}\, \dfrac{1}{r} = \dfrac{i\lambda\pi/2}{r+\lambda\pi}\, \hat{u}^{+}\partial_i \hat{u} \\[2ex] \quad '' \qquad \hat{u}^{+}\dfrac{1}{r^2} = \dfrac{i\lambda\pi/2}{r+\lambda\pi}\, \hat{u}^{+}/r \end{cases}$$

$$\partial_i A_j = i f\, \partial_i \hat{u}^{+} \partial_j \hat{u} + i\, \partial_i f\, \hat{u}^{+} \partial_j u\,, \qquad f \equiv \frac{\lambda\pi/2}{r+\lambda\pi}$$

$$[A_i, A_j] = -[f\, \hat{u}^{+}\partial_i u,\, f\, \hat{u}^{+}\partial_j \hat{u}]$$

$$= +f^2\, \partial_i \hat{u}^{+} \partial_j \hat{u}$$

$$\partial_i A_j - i[A_i, A_j] = i f(1-f)\, \partial_i \hat{u}^{+} \partial_j u + i\, \partial_i f\, \hat{u}^{+} \partial_j u$$

$$\partial_i \hat{u}^{+} \partial_j \hat{u} = (\sigma_i - \sigma\cdot\hat{x}\,\hat{x}_i)(\sigma_j - \sigma\cdot\hat{x}\,\hat{x}_j)\,/\,r^2 - (i \leftrightarrow j)$$

$$= (\sigma_i \sigma_j - \sigma_i\, \sigma\cdot\hat{x}\, \hat{x}_j - \sigma\cdot\hat{x}\, x_i \sigma_j)\,/\,r^2$$

$$= (2i\, \varepsilon_{ijk}\, \sigma_k - 2i\varepsilon_{ik\ell}\, \sigma_\ell\, \hat{x}_k \hat{x}_j)\,/\,r^2 = 2i\, \varepsilon_{ijk}\, \hat{x}^k\, \sigma\cdot\hat{x}/r^2$$

$$\hat{u}^{+}\partial_j \hat{u} = \sigma\cdot\hat{x}\,(\sigma_j - \sigma\cdot\hat{x}\,\hat{x}_j)/|x| = i\,\varepsilon_{\ell jk}\, \hat{x}_\ell\, \sigma_k\, /\, r$$

$$\partial_i f = -\frac{\lambda\pi/2}{(r+\lambda\pi)^2}\, \hat{x}_i$$

$$\partial_i f\, \hat{u}^{+}\partial_j \hat{u} = -\frac{\lambda\pi/2}{(r+\lambda\pi)^2}\cdot i\, \hat{x}_i\, \varepsilon_{\ell jk}\, \hat{x}_\ell\, \sigma_k\, /\, r$$

$$= \frac{i\lambda\pi/2}{(r+\lambda\pi)^2}\cdot \hat{x}_i\, \varepsilon_{j\ell k}\, \hat{x}_\ell\, \sigma_k\, /\, r$$

$$f(1-f) = \frac{\lambda\pi/2\,\left(r+\frac{\lambda\pi}{2}\right)}{(r+\lambda\pi)^2}$$

This solution has both mag. & electric fields.

The mag. part contains the t'Hooft monopole-like field. I suspect that this solution is equiv. to the Zee-Julia solution? No.

$$U = \exp[i\int A\cdot dx]$$

$$\Rightarrow \quad \frac{d}{ds}U = i A\cdot\dot{x}\, U$$

$$\frac{d^2}{ds^2}U = [i(A\cdot\ddot{x} + \dot{A}\cdot\dot{x}) - (A\cdot\dot{x})^2]\, U$$

Now let $A_\mu = i f u^+ \partial_\mu u \Rightarrow i A\cdot\dot{x} = -f u^+\dot{u}$

$$\frac{d}{ds}(iA\cdot\dot{x}) = -\dot{f} u^+\dot{u} - f\dot{u}^+\dot{u} - f u^+\ddot{u}$$

$$(iA\cdot\dot{x})^2 = f^2 u^+\dot{u}\, u^+\dot{u} = -f^2\dot{u}^+\dot{u}$$

so $\frac{d^2}{ds^2}U = [-\dot{f} u^+\dot{u} - f(1+f)\dot{u}^+\dot{u} - f u^+\ddot{u}]\, U$

$$\dot{U} = -f u^+\dot{u}\, U = f\dot{u}^+ u\, U$$

$U = u^+ V \rightarrow \quad f\dot{u}^+ u\, U = f\dot{u}^+ V$

$\dot{U} = \dot{u}^+ V + u^+\dot{V} \qquad \dot{V} = (fu\dot{u}^+ - u\dot{u}^+)V = (f-1)u\dot{u}^+ V$

$= (1-f)\dot{u} u^+ V$

Symmetry $f \leftrightarrow 1-f$, $u \leftrightarrow u^+$

$$u^+\dot{u} + \dot{u}^+ u = 0 \rightarrow 2\dot{u}^+\dot{u} + u^+\ddot{u} + \ddot{u}^+ u = 0$$

This approach does not give you anything. I think one has to consider random walk problem.

One dimensional random walk:

$P_n(x)$: probability of reaching point x in n steps.

$$\cancel{P_{n+1}(x)} = \frac{w}{2}\Big(P_n(x+\Delta) + P_n(x-\Delta)\Big) - P_n(x)$$

$P_{n+1} - P_n(x)$

$$\rightarrow \frac{w}{2}\Delta^2 \frac{\partial^2}{\partial x^2} P_n$$

or $\dfrac{\partial P}{\partial n} = \dfrac{w}{2}\Delta^2 \dfrac{\partial^2}{\partial x^2} P$ diffusion eq.

For N-dimensions, one has the N-dim. Laplacian.

Randome walk in group space:

Displacements in group space are generated by

$$\exp[\sum \omega_a M_a] \qquad M_a : \text{generators}$$

$$= 1 + \omega \cdot M + \frac{1}{2}(\omega \cdot M)^2 + \cdots$$

Averaging over all directions $\rightarrow 1 + \frac{1}{2}\langle(\omega \cdot M)^2\rangle + \cdots$

$$= 1 + \frac{1}{2}\underbrace{\langle \omega_a^2 \rangle}_{\substack{\text{III} \\ \omega^2}} \sum M_a^2$$

Thus the Casimir inv. $\sum M_a^2$ shows up. $= -\vec{L}^2$

↳ ang. mom.

If I apply this to $U[\sigma]$: we find $\omega^2 = \langle A_\mu^2 \rangle$

But this is gauge dependent.

$x_\mu \rightarrow x_\mu + \Delta$ is governed by $A_\mu(x)$

$x_\mu + \Delta \rightarrow x_\mu$ " $A_\mu(x_\mu + \Delta)$

] two pts are not completely equivalent

The inconsistency is avoided by demanding $\partial_\mu A_\mu = 0$ in an isotropic medium.

Suppose $\langle U[\sigma] \rangle \sim e^{-\mu r}$, how does one arrive at a linear potential picture?

The "propagator" for U: $\langle U(t+\delta t)^{+} U(t) \rangle$

if this $\sim \exp[-i\mu^2 r \delta t] / \cancel{\delta t}$, Then it looks like

$\sim e^{-iV\delta t}$, $V = \mu^2 r$

It is necessary that a proper renormalization (subtraction) is made: $\langle U^{+}(t)\, U(t) \rangle = 1$

We know that $\dfrac{\delta U}{\delta x_0} = i \int ds\, (U, F_{t0})$

This observation ~~is~~ amounts to repeating the result that $F_{t0} \sim$ const. is necessary for string formation.

Take a look at the classical trajectory

$$L = \int C(\dot{x})^{2n}\, d\tau + \int \dot{x}_\mu A_\mu \, d\tau \qquad \text{with arbitrary } n.$$

If $A=0$: $\quad p_\mu = 2n\, C(\dot{x}^2)^{n-1}\dot{x}_\mu$

$$p^2 = 4n^2 C^2 (\dot{x}^2)^{2n-1} \qquad \dot{x}^2 = \left[\frac{1}{4n^2C^2}p^2\right]^{\frac{1}{2n-1}}$$

$$\dot{x}_\mu = p_\mu \Big/ 2nC\left(\frac{1}{4n^2C^2}p^2\right)^{\frac{n-1}{2n-1}}$$

$$\mathcal{H} = p\cdot\dot{x} - L = 2nC(\dot{x}^2)^n - C(\dot{x}^2)^n \cancel{- \dot{x}\cdot A}$$

$$= (2n-1)\, C \left[\frac{1}{4n^2C^2}p^2\right]^{\frac{n}{2n-1}} \cancel{- \dot{x}\cdot A}$$

$$= \frac{(2n-1)\, C}{(4n^2C^2)^{n/2n-1}}\, p^{2\,\frac{n}{2n-1}} = \frac{(2n-1)\, C^{-1/2n-1}}{(4n^2)^{n/2n-1}}\, p^{2\,\frac{n}{2n-1}}$$

$A \neq 0$: $\quad p_\mu = 2n\, C(\dot{x}^2)^{n-1}\dot{x}_\mu + A_\mu \quad$ so replace $p \to p_\mu - A_\mu$ as usual.

Now if we want Klein-Gordon form, $\frac{n}{2n-1} = 1 \Rightarrow n = 1$

If $\mathcal{H} \sim (p^2)^2 \to \frac{n}{2n-1} = 2$, $\quad n = \frac{2}{3}$

For $n = 1/2$, $\quad \underbrace{\mathcal{H} \sim (p^2)^{\infty}}_{0\times}$ this is the scale inv. L.

Add a mass term to L : $-\int m\, d\tau$

Eq. for $x(\tau)$:

$$\dot{p}_\mu - \dot{x}_\lambda \frac{\partial A_\lambda}{\partial x_\mu} = 0$$

or $$\frac{d}{d\tau}\left[2n\, C(\dot{x}^2)^{n-1}\dot{x}_\mu\right] + \underbrace{\dot{A}_\mu - \dot{x}_\lambda \frac{A_\lambda}{\partial x_\mu}}_{} = 0$$

$$\left(\frac{\partial A_\mu}{\partial x_\lambda} - \frac{\partial A_\lambda}{\partial x_\mu}\right)\dot{x}_\lambda = F_{\lambda\mu}\dot{x}_\lambda$$

$\Rightarrow$ $\dot{x}^2$ = const. so classical trajectories are indep of n.

for a fixed $\dot{x}^2$