Nov. 1978 — May 1979

THE UNIVERSITY OF CHICAGO

1892

UNIVERSITY OF CHICAGO BOOK STORE

An extension of functional integration formalism.

$$\int \exp[-I]\, D[\varphi] \qquad \text{in Euclidean field theory}$$

$$I = \bar{L}$$

This is analogous to a partition function in 4 dimensions

But $D[\varphi]$ is not a phase space in the Hamiltonian sense.

So let (φ, π) be a canonical pair. I is a functional of φ only. The phase space $D[\varphi]D[\pi]$ only introduces an extra infinite factor.

Now in quantum sense, $\pi(x) = -i\,\delta/\delta\varphi(x)$.

Note that the operator $\pi(x)$ appears in the Euler derivation.

The classical sol. corresponds to

$$[\pi, \overset{\bar{L}}{I}] = 0. \qquad \text{or} \qquad \pi \sim 0.$$

We can supplement $\overset{\bar{L}}{I}[\varphi]$ with an arbitrary functional $K(\pi)$ and consider

$$\exp[-\overset{\bar{L}}{I}[\varphi] - \bar{K}[\pi]]$$

Again, it splits into a product

$$\int \exp[-\overset{\bar{L}}{I}]\, D[\varphi] \times \int \exp[-\bar{K}]\, D[\pi]$$

in the "classical" partition function.

Consider $\bar{I} + \bar{K}$ as a "Hamiltonian" $\mathcal{H}$

We might think of a 5th dimension τ, so that

$$\frac{d\varphi}{d\tau} = \frac{\partial \mathcal{H}}{\partial \pi} \quad , \quad \frac{d\pi}{d\tau} = -\frac{\partial \mathcal{H}}{\partial \varphi}$$

The classical sol. corresponds to

$$\frac{d\pi}{d\tau} = 0$$

τ is like the proper time.

E.g.
$$L = \frac{1}{2}\left(\partial_\mu \varphi \, \partial_\mu \varphi + m^2 \varphi^2\right)$$
$$K = \frac{1}{2}\pi^2$$

$$\Rightarrow \quad \left.\begin{aligned} \mathring{\pi} &= + \Box \varphi - m^2 \varphi \\ \mathring{\varphi} &= \pi \end{aligned}\right\} \Rightarrow \quad \ddot{\varphi} = (\Box - m^2)\,\varphi$$

Indeed it is like a 5-dim. theory.

Now go to a super-Lagrangian $\mathcal{L}$ from $\mathcal{H}$

$$\mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\left(\partial_\mu \varphi \, \partial_\mu \varphi + m^2 \varphi^2\right)$$

"Quantum" mechanical partition function.

Eigenvalues of H:

$$\omega_k = (k^2+m^2)^{1/2}$$

$$\ln Z = \sum \ln Z_k \qquad \ln Z_k = \ln(1-e^{-\omega_k}) + \tfrac{1}{2}\omega_k$$

We need a new constant β: $e^{-\beta\omega_k}$

Green's functions:

$$H \to H + j(x)\varphi(x) \qquad \text{or} \qquad j(k)\varphi(-k)$$

Displaced oscillator $\sum_k \left\{ \tfrac{1}{2}\varphi(k)\varphi(-k) + j(k)\varphi(-k) \right\}$

$$= \tfrac{1}{2}\sum(\varphi(k)+j(k))(\ )^+ - \tfrac{1}{2}\sum_k j(k)j(-k)$$

So $\ln Z_k \to \ln\left(1 - e^{-\beta\omega_k \cancel{+\beta\frac{1}{2}|j(k)|^2}}\right) + \tfrac{1}{2}\beta(\omega_k - \tfrac{1}{2}|j(k)|^2)$

$$\delta \ln Z_k = \cancel{\frac{e^{-\beta\omega_k}}{1-e^{-\beta\omega_k}}\left(-\frac{\beta}{2}|j(k)|^2\right)} - \tfrac{1}{4}\beta|j(k)|^2$$

Correction: we should write

$$\tfrac{1}{2}\omega_k^2|\varphi(k)|^2 + j(k)\varphi(-k) \to \tfrac{1}{2}\omega_k^2\left|\left(\varphi(k)+\frac{j(k)}{\omega_k^2}\right)\right|^2 - \frac{1}{2}\frac{|j(k)|^2}{\omega_k^2}$$

so $\delta \ln Z_k = \cancel{\frac{e^{-\beta\omega_k}}{1-e^{-\beta\omega_k}}\left(-\frac{\beta}{2}\frac{|j(k)|^2}{\omega_k^2}\right)} - \frac{\beta}{4}\frac{|j(k)|^2}{\omega_k^2}$

$\beta \to 0$: $\to \cancel{-\frac{1}{2}\frac{|j(k)|^2}{\omega_k^3}} \quad + \quad "$

$$\cancel{\frac{e^{-x}}{1-e^{-x}} + \frac{1}{2} = \frac{1}{2}\frac{e^{-x}+1}{1-e^{-x}} = \frac{1}{2}\coth\frac{x}{2}}$$

~~No change in the partition fu~~

The previous page is wrong.

We compute

$$\mathrm{Tr}\ \varphi(x)\,\varphi(y)\,e^{-\mathcal{H}} \rightarrow \mathrm{Tr}\ \varphi(k)\,\varphi(-k)\,e^{-\mathcal{H}}$$

$$\mathcal{H} = \frac{1}{2}\left[\omega_k^2\,\varphi(k)\,\varphi(-k) + \beta^2\,\Pi(k)\,\Pi(-k)\right]$$

$$\rightarrow \quad \beta\omega_k\left(n_k + \tfrac{1}{2}\right)$$

$$\varphi(k) \rightarrow \frac{\beta}{\omega_k}\,Q(k)$$

So $\quad \langle n|\varphi(k)|n\pm1\rangle\langle n\pm1|\varphi(-k)|n\rangle$

$$= \begin{cases} n+1\ (\beta/\omega) \\ n\ (\beta/\omega) \end{cases}$$

$$\sum = \sum_{n=0}^{\infty} e^{-n\beta\omega}(2n+1) =: \sum_{n=0}^{\infty} x^n\,(2n+1), \qquad x = e^{-\beta\omega}$$

$$= \frac{1}{1-x} + \frac{2x}{(1-x)^2} = \frac{1+x}{(1-x)^2}$$

$$\sum e^{-n\beta\omega} = \frac{1}{1-x}$$

So $\quad \langle\varphi\varphi\rangle = \frac{\beta}{2\omega}\,\frac{1+x}{1-x} = \frac{1}{2}\left(\coth\frac{1}{2}\beta\omega\right)\frac{\beta}{\omega}$

$\beta \to 0: \quad \rightarrow \frac{1}{\omega^2}$

Analyticity: $\Delta(k) = \frac{1}{2}\frac{\beta}{\omega_k}\frac{1+e^{-\beta\omega_k}}{1-e^{-\beta\omega_k}}$

$$\omega_k = \sqrt{k^2+m^2}$$

~~pot~~ singularities $\omega_k = 0, \quad \beta\omega_k = 2n\pi i$

$$k^2+m^2=0 , \quad k^2+m^2+\left(\frac{2\pi}{\beta}\right)^2 n^2 = 0$$

put $k^2 = -x-i\varepsilon$ $\qquad \omega_k = \sqrt{m^2-x-i\varepsilon}$

$$\to -i\sqrt{x-m^2+i\varepsilon} \quad \cancel{\sim i\sqrt{x-m^2}}$$

$$x = m^2+\left(\frac{2\pi}{\beta}\right)^2 n^2 + \eta$$

$$\omega_k = -i\sqrt{\left(\frac{2\pi}{\beta}\right)^2 n^2 + \eta + i\varepsilon} \sim -\frac{2\pi}{\beta}|n| i \left(1+\frac{1}{2}\left(\frac{\beta}{2\pi}\right)^2\frac{1}{n^2}(\eta+i\varepsilon)\right)$$

$$\sim -\frac{2\pi}{\beta}|n| i + \varepsilon - i\eta$$

$$e^{-\beta\omega} = e^{-\beta(\varepsilon-i\eta)} \cong 1-\beta(\varepsilon-i\eta)$$

$$\Delta(k) \sim \frac{1}{2}\frac{\beta^2}{-2\pi n i}\cdot\frac{2}{\beta(\varepsilon-i\eta)} = -\frac{\beta}{\pi|n|}\frac{1}{\eta+i\varepsilon}$$

Residue $= -\frac{\beta}{\pi|n|}$

We have

$$\cancel{\Delta(x) = \frac{1}{m^2-x} + \sum\frac{c_n}{x_n-x} + \int}$$

~~$x_n =$~~

General x: $\Delta(k) = \frac{\beta}{-2i\sqrt{x-m^2+i\varepsilon}}\,\frac{e^{-\frac{\beta}{2}i\sqrt{x-m^2+i\varepsilon}} + e^{+\frac{\beta}{2}i\sqrt{\ }}}{e^{-\frac{\beta}{2}i\sqrt{\ }} - e^{+\frac{\beta}{2}i\sqrt{\ }}}$

$$= -\frac{\beta}{2\sqrt{\ }}\cot\left(\frac{\beta}{2}\sqrt{\ }\right)$$

This formalism does not yield convergence factors; it makes divergence worse: $\frac{1}{\omega^2} \to \frac{1}{\omega}$ for large ω.

No surprise: it is a 5 dim. field theory.

But why should it go like $1/\omega$ for large ω?

The property is independent of dimension. $\sim \frac{1}{\omega}$ because it is an "equal time" correlation $= \int \frac{dk_0}{k^2+m^2} \sim \frac{1}{k}$.

Dec. 4. Studies on stress tensor

Suppose the gauge fields separate in two phases across a boundary., as some people claim. Is it really possible?

In superconductive analogy, the mag. field balances against Higgs.

Monopoles = Higgs?

One needs a description of monopoles &/or instantons.

If there is a boundary between two phases, magnetic & electric, the pressures must balance. The stress tensor is indeed symmetric:

$$T_{ij} = E_i E_j + B_i B_j - \frac{1}{2}(\vec{E}^2 + \vec{B}^2)\delta_{ij}$$

$T_{ii} > 0$ means tension

< 0 pressure

So $T_{xx} = \cancel{\frac{1}{2}(E_y^2 + E_z^2) + \frac{1}{2} E_x^2 + (E \leftrightarrow B)}$

$\frac{1}{2}(E_x^2 - E_y^2 - E_z^2) + (E \leftrightarrow B)$

The fields must be parallel to the boundary, and

$$E_{\parallel}\big|_1 = B_{\parallel}\big|_2$$

But for free Y.M. fields. There are no real monopole solutions.

In the case of instantons, the pressure is zero:

$T_{ij} = 0$ because $E = iB$. $(ij = 1, \cdots 4)$

This should be so because it is a self-sustaining solution.

If $\exists$ pressure, it must come from entropy!

In the 4-dim view, $-L$ is the "energy" $= T_{55}$

~~So~~ and $T_{\mu\mu} = 0$, $T_{5\mu} = 0$

Since $T_{55} \neq 0$ for instantons, the system is not massless?

There is something strange.

$$T_{\alpha\beta} = F_{\alpha\gamma} F_{\gamma\beta} - \frac{1}{4} g_{\alpha\beta} F_{\gamma\delta} F_{\gamma\delta}$$

$\partial_\alpha T_{\alpha\beta} = 0$ This is true for any dimension.

But $T_{\alpha\alpha} = F \cdot F \left(1 - \frac{n}{4}\right)$!

This also tells that a sol. $\neq 0$ exists only in 4 dim.

The .