A New Look at Fluid Dynamics

Newton dynamics of mass a points may be regarded a mapping of a point in 3-D space to another point. String theory in its orignal form is a mapping of a line segment in 3 space to another line segment. ~~in space~~.

Similarly, fluid motion may be regarded as a continuous mapping of n-D onto itself.

Incompressibility may be imposed as a susidiary condition on ~~for~~ the Jacobian ~~of Jacobian~~ ~~of~~ for the ~~the~~ change of volume elements.

$\cancel{L = \int \frac{1}{2} m}$ ~~Original coordinates of points~~ $\cancel{(\xi, \eta, \zeta)}$

~~Present cordinates~~ $\cancel{(x, y, z)}$

$L = \int \cancel{\frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \lambda}$

In this formulation, the ~~multiplication~~ factor $\lambda(\xi,\eta,\zeta)$ ~~in the con~~ multiplying the condition turns out to be the pressure.

$$L = \int \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x,y,z)$$
$$+ \lambda\left([x,y,z]_{\xi\eta\zeta} - 1\right) d\xi\, d\eta\, d\zeta$$

$$[x,y,z]_{\xi\eta\zeta} \equiv \frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)} \qquad (1)$$

Eqs of motion

$$m\ddot{x} + \partial_x V + \partial[\lambda,y,z]$$
$$= [x,y,z]_{\xi\eta\zeta} \times [\lambda,y,z]_{xyz}$$
$$= [\lambda,y,z]_{xyz} = \partial_x \lambda \qquad (2)$$

etc.

~~Incompressibility implies~~

Fist consider 2D, $[x, y, z] \to [x, y]_{xy}$ Incompressibility implies

" $\mathrm{div}\, v = 0$. ~~for a~~

Since This is a 1st class constraint, To satisfy

this, one this, one ~~assumes~~ introduces the stream function ϕ

$$\dot{x} = \partial_y \phi, \quad \dot{y} = -\partial_x \phi \qquad (2)$$

~~ϕ is a stream function.~~

$$\text{or} \quad \dot{x} = [x, \phi] \quad \dot{y} = [y, \phi] \qquad (3)$$

~~ϕ~~ For a steady stream, $\partial_t \phi = 0$

(3) ~~This~~ has the same form as the Hamiltonian

equation, with x, y ~~being~~ the canonical pair q, p, being equivalent to

$[a, b]$ ~~being~~ is the Poison bracket, and

ϕ playing the role of the Hamiltonian.

In this way one can think of space quantization

In 3-D, one introduces two scalars ϕ, χ

and write

$$\dot{x} = \frac{\partial(x, \phi, \chi)}{\partial(\xi, \eta, \zeta)} \equiv [x, \phi, \chi]$$

In general

$$\dot{O} = [O, \phi, \chi] \qquad (4)$$

($\phi \vec{\nabla} \chi$ is ~~called~~ equivalent to the Clebsch potential)

Similarly for n-D, we introduce $n-1$ Hamiltonians

$$\dot{O} = [x, \phi, \chi, \cdots] \qquad (5)$$

This is the so-called ~~Nambu~~ Nambu's many-Hamiltonian formalism.

The physical meaning of $\phi, \chi, \cdots$ is the following

ϕ = consts in n-D is a $(n-1)$-D manifold.

The intersection of all $\phi, \chi, \cdots$ is a ~~1-D line~~ point.

$\nabla\vec{C}\cdot\nabla\times C = (\nabla\vec{C}\times\nabla)C$

3D compatibility

$\vec{v} = \vec{\nabla}\times\vec{C}$ $\qquad \dot{O} = (\nabla()) \cdot (\nabla\times C)$

$\dot{v} = \nabla(\nabla\times C)\cdot(\nabla\times C)$

$\nabla\times\dot{v} = 0 \qquad \nabla\times\nabla\cdot(\nabla\times C)\cdot(\nabla\times C) = 0$

$-(\nabla\cdot(v\times C)\times\nabla(v\times C) + \nabla(\nabla\times(v\times C))(v\times C)$

$(v\times C)\cdot\nabla(\nabla\times(v\times C))$

$-(v\times C)\cdot\nabla\times\nabla\cdot(v\times C)$

$\cancel{-((v\times C)\times\nabla)\cdot(\nabla\cdot(v\times C))}$

$\cancel{-(v\times C)\cdot\nabla}$

$\cancel{(v\times C)_i\,\nabla_i}\ \nabla_{jk}(v\times C)_i$

$\dot{v}_i \qquad \nabla_{k}(\nabla\times C)_i\cdot(\nabla\times C)_k$

$(\nabla\times\dot{v})_{jl} \qquad \nabla_{jl}\nabla_k(\nabla\times C)_i(\nabla\times C)_k \overset{\nabla}{=} \cancel{(\nabla\times C)_{ijl}}$

$= \nabla\nabla_{jl}(\nabla\times C)_i\,\cancel{(\nabla\times C)}(\nabla\times C)_k$

$= \nabla_{jl}\nabla_k(\nabla\times C)_i\cdot(\nabla\times C)_k + \nabla_{jl}\cancel{(\nabla\times C)_i\ \nabla_{k}(\nabla\times C)_k}$

$+ (\nabla_k(\nabla\times C)_i)\nabla_{jl}(\nabla\times C)_k$

$\nabla_{jl}(\nabla\times C)_i\nabla_k(\nabla\times C)_k = 0$

$+\vec{\nabla}\nabla\cdot\dot{C}$

$(\nabla\times C)\cdot\nabla\nabla\times(\nabla\times C) = \cancel{(\nabla\times C)\nabla\Delta C}\ \cancel{\Delta C\ \nabla\times\Delta} = [\nabla\times(\nabla\times C)]^{\cdot} \sim (\Delta\vec{C})^{\cdot}$

$\cancel{(\nabla\times C)\nabla\Delta\vec{C}} = \nabla\cdot(C\times\Delta\dot{C})$

$\nabla\times C = \nabla\times(\nabla\phi\times\nabla\chi) = \nabla^2\phi\,\nabla\chi - \nabla\phi\,\nabla^2\chi$

$\cancel{\nabla\times(\nabla\times C)} \quad (\Delta C)^{\cdot} = 0 \Rightarrow \Delta\vec{C} = \vec{F}[\phi,\chi]$

$[F,\phi,\chi] = F_\phi[\phi,\chi] + \dots$

$+ F_\chi[\chi,\phi,\chi]$

$$\nabla\times\dot{v} = \vec{\nabla}\times\left((\nabla\times C)\cdot\cancel{\nabla(\nabla\times C)}\,\vec{\nabla}\,\overrightarrow{(\nabla\times C)}\right)$$

$$= (\nabla\times\dot{C})\cdot\nabla\left(\nabla\times(\nabla\times C)\right)$$

$$+\ (\nabla\times\vec{\nabla}C\cdot\nabla)\times\overrightarrow{(\nabla\times C)}$$

$$\nabla\cdot(\nabla\times C)$$

$$\cancel{(\nabla\cdot(\nabla C\times\nabla)}$$

$$\vec{\nabla}\nabla\cdot(C\times\nabla)\times\overrightarrow{(\nabla\times C)}$$

$$\cancel{\nabla\cdot(\vec{\nabla}C\times\nabla)}\ ($$

$$\nabla\cdot(\vec{\nabla}C\times\nabla)\times\overrightarrow{\nabla\times C}$$

$$(\nabla\times\nabla\vec{C}\cdot\nabla)\times(\nabla\times C)$$

$$\nabla\times A\cdot\nabla = \nabla\cdot(A\times\nabla)$$

$$\nabla\nabla A\ \partial_j\ a$$

$$0 = \nabla\times\dot{v} = \nabla\times(\nabla\times\dot{C}) = -\nabla^2\dot{C} + \nabla(\nabla\dot{C}) \cancel{=\nabla\cdot(}$$

$$\cancel{\nabla\phi\times\nabla\chi}$$

$$\phi$$

$$\nabla^2 C = \phi$$

$k = \cancel{1.38 \times 10^{-23}}$ 8.62×10^{-5} eV $K^{-1} \sim 10^{-4}_{eV} K^{-1}$

$M_p c^2$ 940 MeV $= 9.4 \times 10^{8}$ eV $\sim 10^{9}$ eV

$kT = 8.62 \times 10^{-5}$ eV $\sim 10^{-4}$ eV, $100^{\circ}(kT) = 8.62 \times 10^{-3}$

$c = 10^{11}$ cm/sec

$G = \cancel{6.6}\ 6 \times 10^{-8} \sim 6 \times 10^{-7}$ cm³/gm $\sim 10^{-30}$ cm³/eV

$1\,gm \sim 10^{24} M_p c^2 \sim 10^{23}$ eV

$$ds = \sqrt{c^2 dt^2 - dr^2} = c\,dt\sqrt{1 - \frac{dr^2}{c^2 dt^2}}$$
$$= c\,dt\left(1 - \frac{1}{2}\frac{1}{c^2}\frac{dr^2}{dt^2} + \cdots\right.$$

$$\rightarrow -mc\sqrt{} = -mc^2 dt + \frac{m\,dr^2}{2\,dt^2} - \cdots$$

$$\rightarrow \sqrt{g_{\mu\nu}\, dx_\mu dx_\nu} : \rightarrow g_{00} dt dt + g_{0i} dt dx^i + g_{ik} dx^i dx^k$$

$$\Delta = \nabla \cdot \nabla \rightarrow \begin{matrix} \text{div}\cdot\text{grad} \\ \cancel{\frac{1}{r}\partial r} \end{matrix} \quad \begin{matrix} \Delta \frac{1}{r} = -\nabla \frac{\vec{r}}{r^3} = -\frac{3}{r^3} + \frac{3r^2}{r^5} = 0 \\ \cancel{\Delta \frac{1}{r} = -\frac{1}{r^3}} \qquad\qquad \sim 0 \end{matrix}$$

$$\int \delta^3(r)\, d^3r = \cancel{\int \delta^3(r) 4\pi r^2 dr = 4\pi} \int \delta^3(\vec{r})\, dx\,dy\,dz = 1$$
$$\oint \text{div grad} \frac{1}{r} d^3r = -\oint \text{div} \frac{1}{r^2} \vec{n}\, = -4\pi$$

Ars Longa, Vita brevis

Ars vitaque longa

芸術は永いが人世も永い

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億

$$\Delta\, kT\rho + 4\pi G m \rho + \Delta\left(\frac{Gm}{2}(x^2+y^2)\rho\right) = 4\pi G M \delta^3(\vec{x})$$

$$X \equiv kT + Gm\frac{1}{2}(x^2+y^2)$$

$$\Delta(\rho X) + 4\pi G m \rho = 4\pi G M \delta^3(r)$$

$$\cancel{4\pi G m X X^{-1}}$$

WKB $\qquad F = \rho X = \left(\exp(\phi)/r \times (4\pi G M \delta^3)\right.$

$$\cancel{\rho X = \exp(\int \ldots)/r \times GM/r}$$

$$\rho X \equiv \phi$$

$$\nabla\phi\cdot\nabla\phi + \Delta\phi = X = kT + \frac{1}{2}\left(x^2+\frac{1}{2}y^2\right)\rho$$

If $\cancel{\phi = \sqrt{kT}\rho + \alpha(x^4+y^4)\rho}$

$$\cancel{\nabla_x\phi = \sqrt{kT}\nabla_x\rho + \alpha x^3\rho + \alpha x^4\nabla\rho}$$

$$\cancel{\nabla_y\phi = \sqrt{kT}\nabla_x\rho + y^3\rho + y^4\nabla\rho}$$

$$\cancel{(\nabla\phi)^2 = (kT + x)\, kT\nabla^2\rho +}$$

$$\cancel{= (\sqrt{kT}+\alpha)^2\, \nabla\rho} \quad kT\nabla^2\rho + \alpha\sqrt{kT}$$

$$\nabla_x F = (\nabla_x\phi)F \cancel{- \frac{x}{r^3}\exp(\;)} + (\nabla_x \tfrac{1}{r})\exp$$

$$\nabla_y F = \left(\nabla_y\phi - \frac{y}{r^2}\right)F \cancel{- \frac{y}{r^3}\exp(\;)} + (\nabla_y \tfrac{1}{r})\exp$$

$$\nabla^2 F = \nabla^2\phi F + 2\nabla_x\phi\,\nabla_x\frac{1}{(r-x_1)}F + 2\nabla_y\phi\,\nabla_y\frac{1}{(r-x_1)}F + \nabla^2\frac{1}{r}F$$

$$2\vec{r}\cdot\nabla\phi + 4\pi\phi\exp(\;)/r$$

$$\phi = 1 + \alpha(x^2+y^2)$$

$F = \exp(\alpha \rho^2)/r$

$\cancel{\nabla_x F = \alpha \nabla_x \rho F + (\exp)(\nabla_x 1/r)}$

$\cancel{\nabla_y F = \alpha \nabla_y \rho F + (\exp)(\nabla_y 1/r)}$

$\cancel{\nabla^2 F = \alpha \nabla^2 \rho F + \alpha^2 \nabla\rho \cdot \nabla\rho F + \alpha \nabla\rho \cdot \nabla \frac{1}{r}(\exp)}$

$\nabla_x F = \alpha \frac{x}{r} F - \exp()\, 2x/r^3$

$\nabla_y F = \alpha \frac{y}{r} F - \exp() \cdot 2y/r^3$

$\cancel{\nabla^2 F = \alpha^2 F + \alpha}$

$\nabla_z F = \alpha z/r F - \exp()\, 2z/r^3$

$\cancel{\nabla_x^2 F = \alpha^2 \frac{x^2}{r^2} F - \frac{2\alpha x^2}{r^3} - \alpha \frac{x}{r} \cdot \frac{2x}{r^3} + 6x^2}$

$\nabla^2 F = (\frac{\partial^2}{r^2} + \frac{2}{r}\frac{\partial_r}{2}) F \qquad F = \exp(\alpha \rho^2)/r$

$\frac{\partial}{\partial r} F = \alpha \cdot \exp(\alpha r)/r - \exp()/r^2$

$\cancel{(\frac{\partial}{\partial r})^2 F = \alpha^2 \exp()/r -}$

$\frac{2}{r}\partial_r F = 2\alpha \exp()/r^2 - 2\exp()/r^3$

$\frac{\partial^2}{\partial r^2} F = \alpha^2 \exp()/r - 2\alpha \exp()/r^2 + 2\exp()/r^3$

$\nabla^2 F = \alpha^2 F$

Let $F = \exp(f)/r$

$\nabla F = f' F - \exp()/r^2$

$\nabla_r^2 F = f'' F + f'^2 F + 2\exp()/r^3 - f' \exp()/r^2$

$\frac{2}{r}\nabla_r F = 2(f'/r) F - 2\exp()/r^3$

$(\nabla^2 + \frac{2}{r}\nabla_r) F = f'' F + f'^2 F + \cancel{\ldots} - \frac{2f'}{r}\exp()/r^2$
$+ \frac{2f'}{r} F$

$\Sigma = (f'' + f'^2) F$

$$f'' + f'^2 = X^{-1} = 1/(kT + \tfrac{1}{2}(x^2+y^2)\Omega)$$

2D case $\quad \Delta f = \partial_r^2 f + \frac{1}{r}\partial_r f$

$\cancel{\partial_r f} = F = \exp(f)\ln r$ $\quad$ This is not what we want

$\cancel{\partial_r f =}$

$F = \exp(f)\cancel{/}\ln r \quad r^2 = x^2 + y^2$

$\partial_x F = f_x F - \frac{x}{r^3}$

$\partial_y F = f_y F - \frac{y}{r^3} \qquad F = \exp(f)$

$\cancel{\partial_z F = f_z F}$

$\Delta F = f_r^2 + \frac{1}{r} f_r$

$\partial_r F = f' F - \exp(\,)/r$

$\partial_r^2 F = (f'' + f'^2)F + \exp(\,)/r^2 - f'\exp(\,)/r$

$\frac{1}{r}\partial_r F = \frac{1}{r} f' F - \exp(\,)/r^2$

$(\partial_r^2 + \frac{1}{r}\partial_r)F = (f'' + f'^2 + \frac{f'}{r})F - f'\exp(\,)/r$

$$f'' + f'^2 + \frac{f'}{r} = X^{-1} = 1/(kT + \tfrac{1}{2}r^2\Omega)$$

If $X^{-1} \cong \frac{1}{kT} - \frac{1}{2}r^2\Omega/(kT)^2$

Let $f = -\frac{r}{\sqrt{kT}} - \alpha \frac{1}{6} r^3 \Omega/(kT)^{3/2}$

$f' = -\frac{1}{\sqrt{kT}} - \frac{\alpha}{2} r^2 \Omega/(kT)^{3/2}$

$f'' = \quad - \alpha r \Omega/(kT)^{3/2}$

$\frac{1}{r}f' = -\frac{1}{r\sqrt{kT}} - \frac{\Omega}{6}\alpha r^2/(kT)^{3/2}$

$f'^2 = \left(\frac{1}{kT}\right) - 2\alpha \frac{r^2}{\Omega}/(kT)^2 + \varepsilon$

$\partial^4 \alpha$

3D にすると

$F = \exp(f)/r$

∂

$\partial_x F = f_x F - \exp()\frac{2x}{r^3}$

$\partial_y F = f_y F - \exp()\frac{2y}{r^3}$

$\partial_z F = f_z F - \exp()\frac{2z}{r^3}$

$\cancel{\partial_x^2+\partial_y^2+\partial_z^2 = \nabla f\cdot\nabla f F - (2r\cdot\nabla f) + \exp(2f)/r^4}$

$\partial_i^2 F = f_{xx}F + f_x^2 F$

$\left(-f_x \exp() \frac{x}{r^3} - 2\exp()\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)\right)$

$\cancel{\Delta F = (\Delta f)F + (\nabla f)\cdot(\nabla f)F - 2\frac{(\nabla f)}{r^3}} + 2\exp()/r^3$

$\cancel{\text{If } f = r\alpha \quad \Delta f = 2\alpha \quad \nabla f\cdot\nabla f = \alpha^2 \quad \frac{2r\cdot\nabla f}{r^3} = 2\frac{f}{r^3} = \frac{2\alpha}{r^2}}$

$2\alpha/r$

$\cancel{\Delta f F = \frac{2\alpha}{r^2}\exp() - \frac{2r\cdot\nabla f}{r^3}\exp() = \frac{-2\alpha}{r^3}\exp()}$

$\Delta F = \Delta f F + \nabla f\cdot\nabla f F - 2(\vec{r}\cdot\nabla f)/r^3 \cancel{+ 2\exp(f)/r^3}$

If $f = \alpha r \rightarrow \Delta f = \frac{2\alpha}{r} \quad (\nabla f)^2 = \alpha^2 \quad \frac{2r\cdot\nabla f}{r^2} = 2\alpha r/r^2$

$\rightarrow \Delta F = \alpha^2 F$ OK

~~If~~ If $\Delta F = [\alpha + \beta(x^2+y^2)]F$

Let $f = a(x) + b(y)$

$\nabla f\cdot\nabla f = a'^2 + b'^2$

$\Delta f = a'' + b''$

$\cancel{\text{Let } a'' + a'^2 = c + dx^2}$ $\qquad b'^2 + b'' = \cancel{e + d}\ \beta y^2 + \gamma$

$\beta x^2 + \gamma$

Let $a' = \beta x \quad a'' = \beta \quad a = \frac{1}{2}\beta x^2, \ 2x\cdot a' = 2\beta x^2$

$\cancel{f = \frac{1}{2}\beta(x^2+y^2) + \beta}$ $\quad 2\beta$

Sum $\quad \frac{1}{2}\beta x^2$

Solve $\quad f'' + (f')^2 - 2x f'/r^2 = \alpha + \beta x^2$

$f' = g \qquad g' + g^2 - 2x\, g/r^2 = \alpha + \beta x^2$

$(g - x)^2 - x^2 + g' = \alpha + \beta x^2$

$g - x = h, \quad h^2 + h' + 1 - x^2 = \alpha + \beta x^2$

$\cancel{1 + h'/h^2 + (1-x^2)/h^2 = \quad 1/h = x}$

$\cancel{1 - x' + x^2(1-x^2) =}$

If $\beta = -1, \quad h^2 + h' = \alpha, \rightarrow h = \pm\sqrt{\alpha} \rightarrow g = \pm\sqrt{\alpha} + x$

$\rightarrow f = \pm\sqrt{\alpha}\, x + \frac{1}{2} x^2$

~~But this is unphysical~~

perturbation. $\quad h_0^2 + 1 - x^2 = \alpha + \beta x^2$

$h_0^2 = \alpha + (\beta+1) x^2$

$h_0 = \pm\sqrt{\alpha + (\beta+1)) x^2}$

$h_0' = \pm(\beta+1) x / \sqrt{\alpha + (\beta+1) x^2}$

small x : $h_0' \simeq (\beta+1) x/\sqrt{\alpha} - (\beta+1) x^3/\alpha^{3/2} \sim \frac{(\beta+1)}{\sqrt{\alpha}} x$

$h \sim \cancel{\sqrt{\alpha + (\beta+1) x^2}} \pm\sqrt{\alpha + (\beta+1) x^2} \mp (\beta+1)\, x/\sqrt{\alpha}$

Directly with g : $\quad g_0^2 = 0$? else $g_0 = 2x, \; g_0' = 2$

$g_0' = 2,$

$h_0 \simeq \pm\sqrt{\alpha} \pm \frac{\beta+1}{\sqrt{\alpha}} x^2 + \frac{(\beta+1) x}{\alpha^{3/2}} + \frac{(\beta+1)^2}{\alpha^3} x^2$

$g \simeq \pm\sqrt{\alpha} + [\frac{(\beta+1)}{\alpha^{3/2}} + 1] x + [\frac{(\beta+1)^2}{\alpha^2} \pm \frac{(\beta+1)}{\sqrt{\alpha}}] x^2$

Again

$$\Delta f + (\vec{\nabla} f)\cdot(\nabla f) - 2(\vec{r}\cdot\nabla f)/r^2 = \alpha + \beta(x^2+y^2)$$

$\cancel{f = \frac{ar}{a} + b(x^2+y^2) + cz^2}$ If $f = ar + br^2$

$\Delta f = \frac{2a}{r} + 6b \qquad \nabla f\cdot\nabla f = (a\frac{\vec{r}}{r} + 2b\vec{r})^2$

$= a^2 + 4b^2r^2 + 4abr$

$-\frac{2(r\cdot\nabla f)}{r^2} = -2(\frac{ar}{r} + 2b\,r^2)/r^2 = -(\frac{2a}{r} + 4b)$

$\text{Sum} = 6b + a^2 + 4b^2r^2 + 4abr - 4b$

If f is 2D. $\qquad \nabla f = a\frac{\vec{r}}{r} + 2b\vec{r}$

$\Delta f = \frac{a}{r} + 4b \qquad \nabla f\cdot\nabla f = a^2 + 4b^2r^2 + 4abr$

$-2(r\cdot\nabla f)/r^2 = -2a/r - 4b$

But there is also a contribution $-4f(f)/r^3$

stay in 2D, which gives a term $-4f(f)/r^2$

Add $\delta f = c\ln r$ $\qquad$ extra terms $\delta\Delta f = \frac{c}{r^2}$

$\nabla f = a\frac{\vec{r}}{r} + b\vec{r} + \frac{c\vec{r}}{r^2} \qquad \delta\,\nabla f\nabla f = \frac{c^2}{r^2} + \frac{2ac}{r} + 2bc$

$\Delta f = \frac{a}{r} + 4b \qquad \delta(-2r\cdot\nabla f/r^2) = -2\frac{c}{r^2}$

Total $\quad \frac{a}{r} + 4b + a^2 + 4b^2r^2 + 4abr - \frac{2a}{r} - 4b + \frac{2ac}{r} + 2bc$

$+ (c^2 - c)/r^2$

$\rightarrow 4b - 2b + 4b^2r^2 + a^2 + 2bc + (c^2-c)/r^2 + \frac{2ac}{r}$

So $a^2 + 2b = \alpha$, $4b^2 = \beta$ $\qquad 2(c^2 - c) = 1$

$c^2 - c = 1 \qquad 2ac = 0$

$\cancel{c = (1 \pm \sqrt{5})/2}$

Reconsider 3D we must

$$\Delta F = X^{-1} F, \qquad X^{-1} = 1/(a - b(x^2+y^2))$$

$$\rightarrow \qquad \sim \frac{1}{a} + \frac{b}{a^2}(x^2+y^2)$$

Put $F = \exp(f)/r$ to produce the central sun contribution

$$\Delta_3 F = (\Delta f + \nabla f \cdot \nabla f - 2\nabla f \cdot \vec{r}/r^2) F$$

But If $\tilde{r} = \sqrt{x^2+y^2}$ $\quad \Delta_3 1/\tilde{r} = +\frac{2}{\tilde{r}^3} - \frac{1}{\tilde{r}^3} = \frac{1}{\tilde{r}^3} \rightarrow \frac{1}{\tilde{r}\tilde{r}^2} = \Big\{$ xtra term on r.h.s.

term in f $\sim c \ln \tilde{r}$, $\quad \nabla f \cdot \nabla f = c^2/\tilde{r}^2$

$\sim \nabla f = c\,\vec{\tilde{r}}/\tilde{r}^2 \quad \nabla f \cdot \nabla f = c^2/\tilde{r}^2 \quad \Delta_3 f \cong c/\tilde{r}^2$

$-2\nabla f \cdot \vec{r}/r^2 = -2c/\tilde{r}^2$

Let $c^2 - 2c = 1$ to match r.h.s.

term in f $\sim b\tilde{r}^2 \quad \nabla f = 2b\vec{\tilde{r}} \quad \nabla f \cdot \nabla f = 4b^2\tilde{r}^2$

$\Delta f = 4b$

cross term $\nabla f \cdot \nabla f \rightarrow 2b\,\tilde{r} \cdot c\,\vec{\tilde{r}}/\tilde{r}^2 = 2bc$

Problem : Applying Thermodynamic formula to 2D in a 3D space is prolematic.

Possible answer : If the system is rotating ~~with~~ as a whole, the velocity $v = \Omega r$

If it is in a thermal state, probability is $e^{-\frac{1}{2}mv^2/kT}$

$= \exp(\frac{1}{2}m\Omega^2 r^2/kT) \rightarrow \exp(-\frac{1}{2}m\Omega^2 r^2/kT)$

How big is Ω : $2\pi/365 \times 10^5$ sec $\sim 2\times 10^{-7}$ sec

$m \sim 10^{12}$ gr ? but it varies